

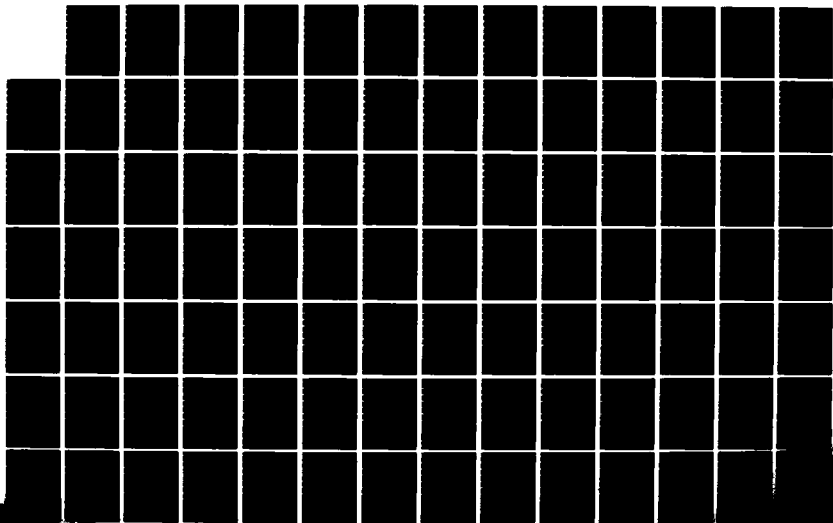
AD-A141 169

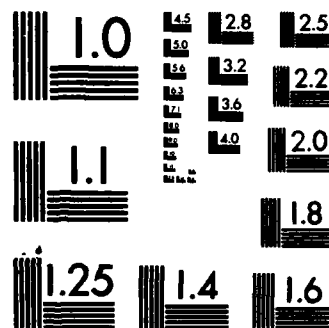
AN OPTIMIZATION METHODOLOGY FOR MULTI--CRITERIA
COMPARISON USING RESPONSE. (U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI... R E GRANEY
MAR 84 AFIT/GST/OS/84M-11 F/G 12/1

1/2

UNCLASSIFIED

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD-A141 169

DTIC FILE COPY

AFIT/GST/OS/84M-11

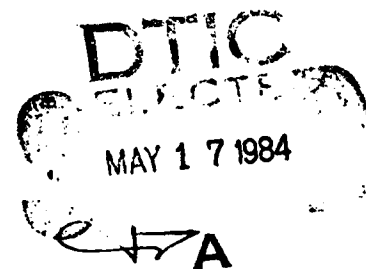
AN OPTIMIZATION METHODOLOGY FOR MULTI-
CRITERIA COMPARISON USING RESPONSE
SURFACE METHODS AND MATHEMATICAL
PROGRAMMING

THESIS

Robert E. Graney
Captain, USAF

AFIT/GST/OS/84M-11

Copy available to DTIC does not
permit fully legible reproduction



This document has been approved
for public release and sale; its
distribution is unlimited.

84 05 15 047

DISCLAIMER NOTICE

**THIS DOCUMENT IS BEST QUALITY
PRACTICABLE. THE COPY FURNISHED
TO DTIC CONTAINED A SIGNIFICANT
NUMBER OF PAGES WHICH DO NOT
REPRODUCE LEGIBLY.**

AFIT/GST/OS/84M-11

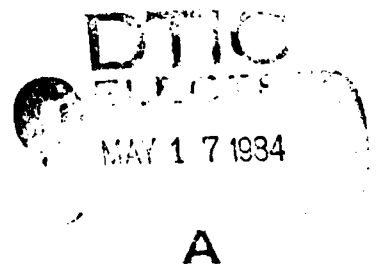
AN OPTIMIZATION METHODOLOGY FOR MULTI-
CRITERIA COMPARISON USING RESPONSE
SURFACE METHODS AND MATHEMATICAL
PROGRAMMING

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Operations Research

Robert E. Graney
Captain, USAF

March 1984



Approved for public release; distribution unlimited

Acknowledgements

First and foremost, I wish to thank my wonderful wife, Susan, for her patience and support throughout this educational endeavor. She has excelled as both typist and quasi-single parent for the past eighteen months.

The basis, guidance, and inspiration for this study came from LtCol Palmer W. Smith who afforded me unlimited time and effort and use of his copyright material for this project. I sincerely appreciate his dedication as my thesis advisor. I also wish to thank LtCol Ivy D. Cook, Jr. as my reader, instructor, and sounding board during this study. His analytical insight has been a great aid in writing this thesis.

My fellow GST students played a large role throughout the educational process and, in particular, Captain James Cooke and Captain Robert Bunnell who gave their invaluable assistance in the construction of the arsenal model.

Finally, I would like to thank Captain Richard Floyd for his help in obtaining the SUMT program and the use of his pamphlet in Appendix D of this material.



A1 23
GP

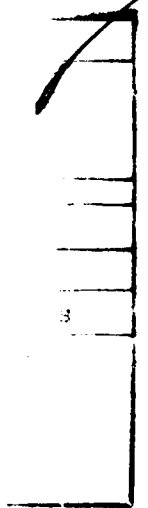


Table of Contents

	<u>Page</u>
Acknowledgements.....	ii
List of Figures.....	v
List of Tables.....	vi
Abstract.....	vii
Chapter One - The Research Problem.....	1
Introduction.....	1
Background.....	2
Literature Review.....	3
Problem Statement.....	17
Research Objectives.....	17
Scope.....	18
General Methodology.....	19
Chapter Two - Methodology.....	20
Introduction.....	20
Theory.....	20
Stepwise Methodology.....	25
Chapter Three - Application Example.....	41
Introduction.....	41
Problem Definition.....	42
Variable Selection.....	45
Control Variable Limits.....	46
Data Source/Generation.....	46
Initial Surface Estimation.....	48
Research Design Selection.....	48
Global Regression Estimate.....	50
Regression Error.....	53
Contour Interval Determination.....	54
Constrained Maxima Determination.....	55
Conclusion.....	55
Chapter Four - Analysis.....	56
Introduction.....	56
Graphic Optimization.....	58
Data Presentation.....	64
Analysis.....	66
Decision Tools.....	74
Conclusion.....	77

	<u>Page</u>
Chapter Five - Concluding Remarks.....	79
Research Summary.....	79
Applications.....	81
Further Research.....	82
Appendix A - Aggregated Arsenal Exchange Model.....	84
Program Create.....	89
MPOS Control Cards.....	94
MPOS Example.....	96
Appendix B - Research Design.....	102
Appendix C - Regression.....	104
SPSS Example.....	104
CF Summary Regression.....	112
AF Summary Regression.....	114
Appendix D - Sequential Unconstrained Mini- mization Technique (SUMT).....	121
Descriptive Pamphlet.....	122
Example Executive Program.....	138
Appendix E - Analysis Data.....	141
CF and CV Contour Data Program "Surface".....	142
Contour Plots.....	143
SUMT Output Tables.....	155
Bibliography.....	164
Vita.....	167

List of Figures

<u>Figure</u>		<u>Page</u>
1.	Example Response Surface.....	11
2.	Response Surfaces' Constrained Maxima.....	22
3.	Methodology Flow Chart.....	26
4.	Contour Plot of CF = 850/860.....	60
5.	Contour Plot of CV = 1176.0.....	60
6.	Overlay Plot of CV = 1176.0 and CF = 850,860...	61
7.	Overlay Plot of CV = 1176.0 and CF = 854.....	62
8.	Overlay Plot of CV = 1176.0 and CF = 856.....	63
9.	Maximum CF subject to CV.....	65
10.	Maximum CV subject to CF.....	65
11.	Maximum CF subject to CV Full Regression Coefficients.....	72
12.	Multiple CF Solution Sets.....	75
13.	Contour Plot of CV = 1175.....	143
14.	Figure 5 Enlargement.....	144
15.	Figure 4 Enlargement.....	145
16.	Contour Plot of CF = 852.....	146
17.	Contour Plot of CF = 854.....	147
18.	Contour Plot of CF = 856.....	148
19.	Figure 6 Enlargement.....	149
20.	Figure 7 Enlargement.....	150
21.	Figure 8 Enlargement.....	151
22.	Figure 9 Enlargement.....	152
23.	Figure 10 Enlargement.....	153
24.	Figure 11 Enlargement.....	154

List of Tables

<u>Table</u>		<u>Page</u>
I.	Weapon System Characteristics.....	43
II.	Target Base Characteristics.....	44
III.	Control Variables' Ranges.....	47
IV.	Research Design and Sampled Values.....	49
V.	Regression Error Summary.....	54
VI.	CF Regression Error.....	115
VII.	CV Regression Error.....	117
VIII.	AF Regression Error.....	119
IX.	SUMT: Maximum CF given CV.....	155
X.	SUMT: Maximum CV given CF.....	157
XI.	SUMT: Maximum CF given AF.....	160
XII.	SUMT: Maximum CV given AF.....	162

Abstract

→ The maxima/minima of one response surface (measure of effectiveness, MOE) is calculated when the domain is constrained by consecutive levels (aspirations) of a second response surface. Thus, the optimal trade-off between the two responses (MOE's) is displayed over an entire operating domain, providing new and unique capabilities to evaluate options.

The study applies this development to a strategic force structure modeled in a linear programming model. The methodology determines the optimal force structure to maximize one MOE given the aspiration levels of the other MOE. Therefore, the decision maker has the optimal force structures to achieve a range of demands/aspirations and also maximize the second goal.

The methodology has applications to multicriteria optimization when relatively large numbers of domain control variable combinations are involved. This technique may be a tool to design the optimal weapon system characteristics or choose the optimal mix of weapon types to fulfill dual mission requirements.

This work is an extension of previous work which used response surfaces, experimental design, and mathematical programming to conduct multi-dimensional, sensitivity analysis and weapon system comparisons.

CHAPTER ONE

The Research Problem

Introduction

This study is concerned with a system's optimal structure of operation when determined by multiple measures of merit. Its purpose is to offer a methodology to aid the multicriteria decision process when involved in a relatively large number of alternatives. In particular, it considers a system's optimization of one measure of merit (response) in comparison to a second measure of merit (response). The research is an extension of Palmer W. Smith's work (29,30) in application of response surface methodology to analytical math modeling.

This first chapter introduces the problem and outlines the study to follow. A brief background and literature review lends further insight to the problem and offers some recommended references for the various methodologies to be studied. With this foundation, the chapter then delineates the problem statement and research objectives. Finally, the scope of the research example is defined. Following the study, the appendices list the model being considered for generating the surfaces, the regression design, the regression analysis, and various samples of computer code employed.

This study involves many disciplines, i.e., research design, response surface methodology, regression analysis, Lagrange multipliers, and non-linear math programming. In

view of this, it is beyond the scope of this paper to attempt an in-depth review of all these disciplines. Therefore, the literature review is primarily concerned with an overview of the current methodologies for multicriteria decision making. The remainder of the literature review is intended more as a descriptive definition of the methodologies that are employed in the study. References to these different methodologies will be made throughout the text and are used during the research.

Background

A system's effectiveness is often determined by more than one measure of merit. These measures of merit are usually determined by the objectives of the system's designer and are not necessarily of the same numeric scale or units of measure (non-commensurate). For instance, a system's commensurate goals may be to increase retail sales and decrease overhead costs. Since both these measures are monetary, the two goals can be combined to a single measure of merit, profit. If, however, the two goals were increased retail sales and decreased production time, then the measure of merit would be money and time (non-commensurate). Obviously, system policies which increase the prior and decrease the latter are sought by the decision maker. However, the actual relationships between the various goals and their respective optimization can, at times, be vague and elusive for the decision maker. Often, the analyst attempts to

transform the different measures of merit (MOE's) into a common unit such as dollars. The transformation, although appealing, may raise some serious questions. Ignizio (15: 375) exemplifies this problem by considering the dilemma of minimizing highway funding and also minimizing highway injuries and deaths. Few decision makers will want the task of trading off dollars for lives. Since a common unit is not always attainable, then the system's operation is characterized by more than one measure of effectiveness (MOE). A vector of multiple MOE's is applied. Each vector element is the level of attainment for one of the decision maker's (system's) objectives, i.e., an MOE. Once the combination of possible MOE's is determined over the operating range, the decision maker must still make a trade-off or a decision as to which is the optimal solution.

Literature Review

In light of the decision maker's problem, several methodologies have been developed to aid in his thought process. Some of the current methodologies for multiple comparisons are multiple goal programming (MGP), multiple attribute utility theory (MAUT), and multiple objectives optimization theory (MOOT).

The first technique, multiple goal programming, is an extension of linear programming and is presented in Ignizio's text (15). Essentially, multiple goal programming sequentially solves a series of linear programs. However,

instead of a single objective function and setting hard values on the right-hand-side of the constraint set, MGP uses deviational variables in the objective function and also the constraint set. Therefore, the sequential linear program attempts to minimize the distance or range of the deviational variables from their right-hand-side values (aspiration levels) (15:376). With this technique, the MGP allows the analyst to sequentially go through a series of lexicographically ordered goals and either minimize or maximize the deviation from the aspiration level set in the right-hand-side value. This technique avoids many of the linear programming cases of infeasibility. Since each MGP objective is made of deviational variables, the algorithm often finds how "close" one can come to the traditional linear programming optimal solution even though the actual aspiration level (right-hand-side) may not be attainable.

However, a disadvantage in the use of the MGP algorithm may occur if the goals require a mix of deviational variables from different right-hand-side values. In this case, the goal in question requires the deviational variables to be commensurate in both common units of measure and equal numeric scale. If a single goal contains non-commensurate deviational variables, the algorithm will treat the unit change in one as equal to a unit change in the other. Thus, if a right-hand-side range for one de-

viational variable were in terms of tenths, whereas a second were in terms of hundreds, the MGP algorithm would be unable to distinguish the deviation of tenths versus that of hundreds. Likewise, it would equally weight the value between unlike measures of effectiveness such as time and money.

Thus, the MGP solution is a lexicographic ordering of the goals based on the deviations from the goals' aspiration levels. It does, as stated earlier, yield a solution even if all goals are not attained. This MGP solution is the "best/optimal" given the order of goals and the relative deviation from the unattained goals. See Ignizio (15:16) for further multiple goal programming discussion.

The next two techniques, MAUT and MOOT, were developed to handle multicriteria decision theory problems. The first, MAUT, is used to elicit from the decision maker his preference function in the trade-off between the system's measures of effectiveness over the range of possible operation. This preference function applies its criteria weight to each alternative policy and then searches for the alternative with the greatest weighted value. This approach requires a good deal of elicitation time with the decision maker and also a calculation of each alternative.

The second approach, MOOT, is a multiple optimization method which attempts to identify a non-dominated solution set (NDSS). The NDSS is the set of solutions or alterna-

tives that cannot be bettered by any other alternatives. This dominance is based on an alternative's set of measures of effectiveness (MOE's). Each alternative's performance is evaluated by its MOE's. Thus, associated with each alternative, A_i , is a set or vector of MOE's, v_i (11:1). The non-dominated solution set is determined, then, by a comparison between each alternative's MOE vector (v_i). When compared to another MOE vector, each element of the non-dominated vector must be greater than or equal to the compared vector and at least one of the non-dominated vector elements must be greater than its respective counterpart in the compared vector. Unlike MAUT, this technique of non-dominance does not require the development of an exacting trade-off between the individual measures of merit which characterize the system's operation. Therefore, the NDSS is an optimal set of alternatives (A_j) with an MOE vector better than any other alternative MOE vector. This solution set is also referred to as a pareto-optimal frontier. Both Keeney and Raiffa (18) and Zeleny (34) offer texts for studying multicriteria decision theory. Dr. Dewispelare's paper (11) offers an excellent review of multicriteria decision theory and also presents a combined MOOT non-dominated solution set and MAUT preference function approach to give the optimal recommendation to the decision maker.

In either case, MOOT or MAUT, the techniques require an appreciable time with the decision maker and also require

his arbitrary choice or trade-off between the measures of merit. Both approaches require a calculation be made upon each possible alternative which, in the case of a very complex or large problem, may take a considerable amount of time.

Besides these three major areas (MGP, MAUT, and MOOT), the literature review also covers a technique employed by the chemical engineering industry. In 1960, Lind, Golden, and Hickman (21) demonstrated the use of a research design to develop a response surface for yield (MOE 1) based on the mix of three possible chemicals (independent domain variables). Their second MOE, cost, was then calculated for the possible combinations of their independent variables. Thus, two response surfaces representing their two measures of effectiveness were created. Each surface was then reduced into iso-value contour lines, similar to those used in typography to define elevations. To then optimize the system, the two sets of contours were then overlaid to find the maximum ratio of yield to costs.

In 1981, Khuri and Conlon (19) showed the optimization of several responses (MOE's) by use of a regression polynomial. In this case, the authors show that they can generate a global "best" solution over multiple response surfaces of the same degree over the same operating domain. This "best" solution is generated from the individual maxima of each response. A distance measure, r , is devised

to move from the original starting point of the individual maxima to a common point of the highest estimated value. This distance function is essentially the distance moved from a maximum divided by the variance of said estimate. Given this distance measure, the analyst may then find the global "best" solution for the multiple MOE's of equal degree response surfaces.

Thus far, the literature review has focused on the area of global optimization. Now the attention is shifted to offering the decision maker a range of optimal solutions for a system's operating domain.

P. W. Smith (29,30) has shown, through the use of orthogonal design and multivariate regression, the ability to accurately predict the increase in a measure of effectiveness due to each of the operating domain elements. He demonstrates the optimal rate of MOE increase per domain element in the orthogonal/independent regression coefficients. Likewise, a pairwise trade-off between domain elements is determined by the ratio of the respective independent regression coefficients. Thus, Smith's regression equation may be used to predict which domain element(s) should be used in the achievement of some aspiration level set by the decision maker. This solves the problem of having to totally rerun a set of linear programs and thus gives the analyst a very powerful tool if the aspiration level should change for the decision maker. This regression

prediction will be good as long as it is used within the original domain limit of the research design. Further use of Smith's study and the extension to this research will be defined in Chapter Two for the proposed methodology.

To adequately review the proposed methodology requires preliminary knowledge of several disciplines such as response surface methodology, multiple regression, and research design. Although each is a subject in itself, a brief definition and overview are offered as follows.

Response Surface Methodology. Response surface methodology was originally proposed by G.E.P. Box and K.B. Wilson in their article in the Journal of the Royal Statistical Society in 1951. Since then, it has been well covered in many texts such as Cochran and Cox (6) and Myers (22). The concept of the response surface is as the name says, a surface such as that of the earth. We may think of a surface as responses or measures of effectiveness that are a function of their domain elements. In the analogy to the earth's surface, we can think of elevation as being the coordinates, latitude and longitude. Thus, the elevation changes as we move across the domain. This surface, then, may be expressed in a functional form where the dependent variable, r , is the measure of effectiveness and is a function of the independent variables, d_k , where k runs from 1 through n , n being the dimension of the domain and $n+1$ being the dimension of the entire surface. Eq (1) is

representative of this functional relationship.

$$r_{ij} = R_i(d_1, d_2, \dots d_n) \quad (1)$$

where i = the type of response considered and j = the domain location (i.e., $j = (d_1, d_2, \dots d_n)$). Eq (1) also demonstrates the response surface's functional relationship for "i" multiple response surfaces over the same domain. Thus, as in our original example, the earth elevation is a measure of merit that would be response one over the domain of latitude and longitude. Likewise, a second response such as the average annual rainfall could also be mapped over the earth's longitude and latitude. This would create a vector of two measures of effectiveness, elevation and average rainfall, as a function of the same domain. This is the same as MOOT's vector of measures of effectiveness used for the non-dominated solution set determination. Figure 1a is representative of a typical response surface where we have two domain elements, d_1 and d_2 . Figure 1a is the geometric representation of a three-dimensional space, and Figure 1b collapses that figure into iso-value, two-dimensional contour lines similar to those used in typography. Each contour line is representative of Eq (1) where the value of r is set at a constant. Thus, we can think of each contour as r_1, r_2, r_3 , and so on. The examples shown are no greater than three-dimensional to allow for graphic illustration. Obviously, however, from Eq (1), we can see that a response surface can be of any dimension depending on the number of

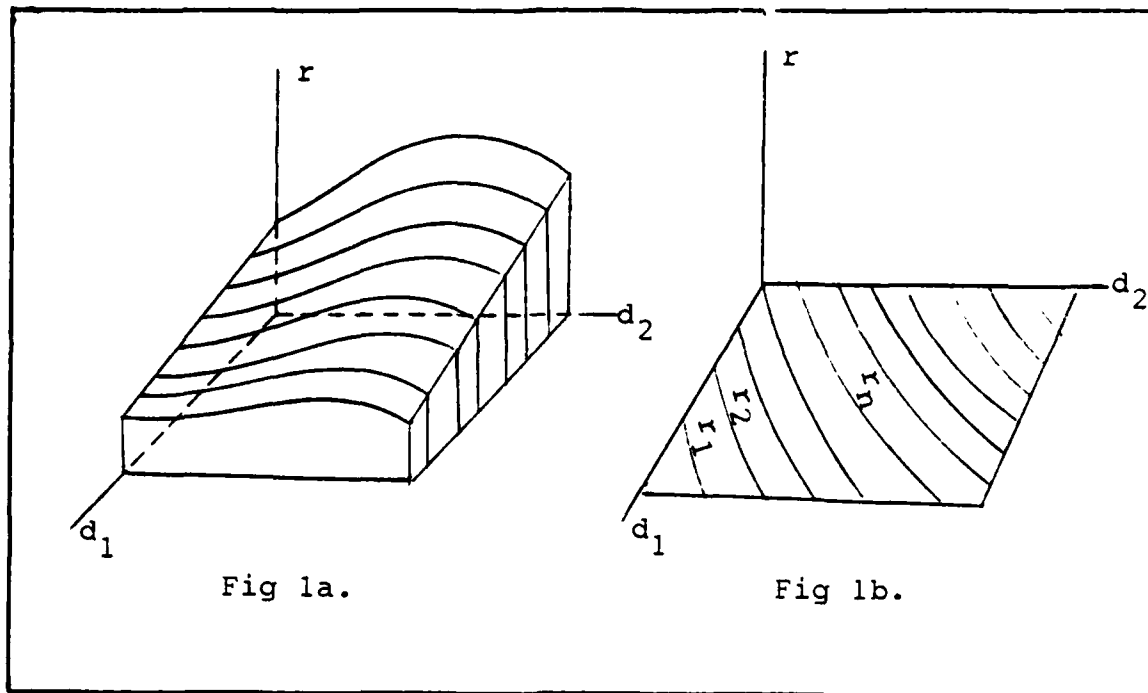


Fig 1. Example Response Surface

variables in our domain. Thus, the purpose of the response surface is to help us visualize, as it were, the functional relationship between our measure of effectiveness and the independent variables in our operating domain. There are also numerous techniques, such as gradient search, for exploring the surface in search of stationary (critical) points. These techniques and further discussion of response surfaces are best covered in Myers' text (22). Next we consider the research design.

Research Design. The research design is a powerful sampling technique used by the experimenter to try to approximate the response surface mentioned above. The number of possible permutations of our domain set can grow very quickly as the range of each domain element increases. Thus, an attempt to evaluate each point may prove tedious if not impossible in some cases. For example, if a three-element domain were to have respective ranges of 10, 15, and 20 units each, then the number of permutations for all combinations would exceed some 3,600 samples. Although this number seems large, consider if the ranges were increased to 20, 40, and 50 respectively. Rather than 3,696 samples, 43,911 samples would be required. Obviously, total enumeration of the surface becomes impossible as the problem's domain range increases. Thus, the analyst must approximate the surface by sampling different points across it. If the surface is known to be flat, the analyst would simply sample each of the corner points and then, connecting these corners, be able to completely define the surface in question. If, on the other hand, the surface is not flat or unknown, this sampling of strictly the corner points is insufficient to describe the changing curvature over the surfaces.

The theory of research design is well documented in numerous texts (6,14,22,32) and also in numerous articles such as Box, et al (3,4). In general, the design is chosen by the experimenter based on some prior knowledge

of the response surface or some assumption about it. Thus, a 2^k design is chosen if the experimenter believes there are no second order terms describing the surface. If, upon testing, his data lacks goodness of fit, the researcher must re-evaluate his original assumption about the curvature of the surface and choose a smaller region to examine or perhaps a higher order model. The 3^k design allows for second order terms in the characteristic equation of the surface and thus would be the next choice. This characteristic equation is the regression of the surface and is discussed in the next definition.

Thus, the research design is nothing more than a choice in the number and location (domain elements' levels) of samples to be taken. The power of the design is its ability to represent the experimental region in as few samples as possible. This power of design is developed by choosing the domain levels (sample locations) such that, when put into an array, the resultant matrix is orthogonal. In this design matrix, each row is a sample and each column is one of the domain elements at its particular level for that sample. Thus, if we had a three-element domain, and ten samples, we would have a ten-row by three-column matrix for our design. Appendix B shows the design matrix applied in this study. The true power of the orthogonal design matrix is in its ability to generate independent coefficients in the general linear regression model mentioned below. To

accomplish this, our design matrix (D) is orthogonal; thus, the transpose times the original matrix equals a diagonal matrix, $D'D = \lambda \cdot I$. There are many published designs which accomplish this (2,3,4, and 8). This study is primarily concerned with a 3^k fractional factorial, rotatable design introduced by Box and Behnken (2). This design will be further discussed in Chapter Two of our methodology.

Multiple Regression. As mentioned earlier, response surface is merely a geometric means to help interpret the functional relationship shown in Eq (1). The researcher chooses the experimental design in hopes of achieving a good characteristic sampling of the surface in question. This characteristic sampling is then expressed in a multiple regression equation. The multiple regression equation is a simplified polynomial expression of a functional relationship depicted in Eq (1). The accuracy of this polynomial lies in its ability to predict points on the surface. Thus, the goodness of fit is determined by the residual error between the polynomial prediction and that of the actual experimental value. This study is primarily concerned with linear regression models rather than the curvilinear regression. DeGroot (9:5-10) and Devore (10:422) both give excellent theoretical descriptions of least square estimates and their use in regression estimates. Likewise, Kleinbaum (20:136) shows regression application to fitting response surfaces. Eq (2) is a

polynomial approximation for the functional relationship in Eq (1). This equation can also be expressed as in Eq (3) in matrix form, where r is a column vector of responses, B is the vector of coefficients, and D is the design matrix as described previously. In minimizing the error term, e , in Eq (3), it has been shown in many texts (9:513, 22:29) that the maximum likelihood estimator which minimizes the variance is B , as shown in Eq (4). The power

$$r = B_0 + B_1 d_1 + B_2 d_2 \dots B_n d_n + B_{11} d_1^2 + B_{22} d_2^2 \dots \\ + B_{nn} d_n^2 + B_{12} d_1 d_2 \dots + B_{(n)(n-1)} d_n d_{n-1} \quad (2)$$

$$r = D \cdot B + e \quad (3)$$

$$B = (D'D)^{-1} D \cdot r \quad (4)$$

of the orthogonal design is again shown in the solution of B estimate. As stated earlier, the power of the $D'D$ term, which collapses into a diagonal identity matrix, is to create the orthogonal polynomials which, in turn, cause each of the coefficients in the B vector to be independent from one another (31:72). The power and use of this independence between the regression coefficients is brought up and used in the methodology and referenced in Smith's work also. Finally, the error term in the regression equation is a measure of our lack of ability to predict the true value on the response surface. This error term is composed of two components, response variance and model bias. Wonnacott

(33:21) gives an excellent example of the error division between bias and variance. In this study, and that of Smith, a deterministic model is used and, as such, there is no stochastic variance in our error term. Thus, designs which minimize the bias induced by the model would be of optimal choice for this study. Karson (16,17) shows the criteria for a minimum bias, two-dimensional design to be orthogonality, rotatability, and equal spacing. However, there is no three-level design which meets the criteria for minimizing only bias. The Box and Behnken design (2) chosen for this study is designed to reduce both measurement variance and model bias.

The last three topics, response surfaces, research design, and regression analysis, have each been briefly reviewed in comparative isolation from one another. In actuality, response surface methodology incorporates all three topics simultaneously. This is evidenced by the inter-relationship between the design level and the order of the regression equation. The three-way interdependence is completed by the regression equation's error term which measures the adequacy of the researcher's estimate of the surface.

The reader should see Myers' "Response Surface Methodology" (22) for a combined, in-depth coverage of these subjects.

Problem Statement

There is a conceptual problem for a system's decision maker between multiple objectives, their respective measures of merit (MOE's), and the optimal system structure/operation to achieve these respective goals. Methodologies mentioned in the literature review aid the decision maker in this process but still depend heavily upon:

- (i) The decision maker's MOE trade-off weight;
- (ii) The use of commensurate goals; and
- (iii) The calculation of each alternative to determine the non-dominated solution set.

Although these problems may not all arise in a particular study, they become more prevalent as the range and number of the domain elements increase. This is exemplified by optimal mix problems mentioned earlier.

Research Objectives

Find the optimal relationship between a system's set of measures of effectiveness over a relatively large operating domain.

Subobjectives:

- (i) Generate the surfaces for comparison over the same domain;
- (ii) Apply a research design to estimate the surfaces in regression equations;
- (iii) Choose response surface, r_1 , to be constrained

by the second response surface, r_2 . Represent the constraining surface, r_2 , in iso-value contour lines;

- (iv) Determine the constrained maxima of r_1 for each of the r_2 contours;
- (v) Map the constrained maxima function for r_1 ; and
- (vi) Reverse the roles of r_1 and r_2 and show the constrained maxima function for r_2 .

Scope

The purpose of this study is to optimally compare multiple response surfaces over the same operating domain. Thus, the development of a highly sophisticated model to generate such surfaces is not deemed necessary. This study uses a deterministic, aggregated, arsenal exchange model to look at the problem of optimal force mix in strategic planning. This model is discussed further in the methodology, Chapter Two, and again in Appendix A.

Since the true functional relationship, Eq (1), is not known, this study assumes a second order relationship between the MOE and the domain variables. This decision was based on the prior work of Smith (30) on similar systems. Smith found the damage expectancy measure of effectiveness to be monotonically increasing and a second order regression polynomial gave excellent results. In view of this prior knowledge, this research will start by using a second

order regression and three-level design.

General Methodology

- (i) Represent one surface in a set of iso-value constraint lines and determine the interval for those lines;
- (ii) Find the constrained maxima of r_1 subject to the iso-constraint of r_2 ; and
- (iii) Map the constrained maximum function in terms of a constraining response.

The next chapter expands the methodology and discusses the specific theory involved. Chapter Three will then apply this proposed methodology to an example. The analysis section of the methodology is covered in Chapter Four.

CHAPTER TWO

Methodology

Introduction

This chapter details the methodology for the comparison and optimization for one response surface to another. It builds on the general methodology described in Chapter One and expands on those areas of the literature review which are pertinent to the proposed methodology.

This methodology consists of 12 steps. Each step is presented both in general format and specific application for Chapter Three. Prior to the actual stepwise presentation, however, an overview of the theoretical basis is covered. This overview serves as the "big picture" for the step-by-step optimization process.

Theory

The theoretical basis for this study is best described by Lagrange multipliers. Protter (27:176) demonstrates the use of Lagrange multipliers to determine the constrained maxima of one function (R_0) over the domain, d_i , subject to the constraints of other functions (R_1, R_2, \dots) over the same domain. Eqs (5) through (9) demonstrate the general methodology for Lagrange multipliers where $R_0 (d_i)$ is the objective to be optimized subject to the requirements of $R_1 (d_i)$.

$$\text{optimize } r_1 = R_0(d_i) \quad (5)$$

$$\text{subject to } R_1(d_i) = 0 \quad (6)$$

The Lagrange multipliers are introduced to form a new function $R'_0(d_i, \lambda_j)$ where d_i is the original domain and λ_j is the multiplier for the j^{th} constraint.

$$\text{optimize } R'_0(d_i, \lambda_j) = R_0(d_i) + \lambda_j \cdot R_j(d_i) \quad (7)$$

Thus, the problem is now to optimize the new function $R'_0(d_i, \lambda_j)$. This is accomplished by simultaneously solving the set of partial derivatives of R'_0 set equal to 0.

$$\text{solve } R'_0/\partial d_i = 0 \text{ for all } i\text{'s} \quad (8)$$

$$R'_0/\partial \lambda_j = 0 \text{ for all } j\text{'s} \quad (9)$$

The solution to this set of equations is the set of critical (stationary) points for R_0 subject to R_j . Protter (27:178) gives several numeric examples of this technique.

Chapter One's literature review describes the response surface graphically in Figure 1, and functionally in Eq (1). The review also describes the researcher's approximation for the response surface functional relationship in a multi-variable regression polynomial, Eq (2). The combination of these techniques with Lagrange multiplier optimization lays the foundation for this study.

Figure 2a depicts the surface to be optimized, $R_0(d_i)$. Figure 2b depicts the constraint surface $R_1(d_i)$ in both the

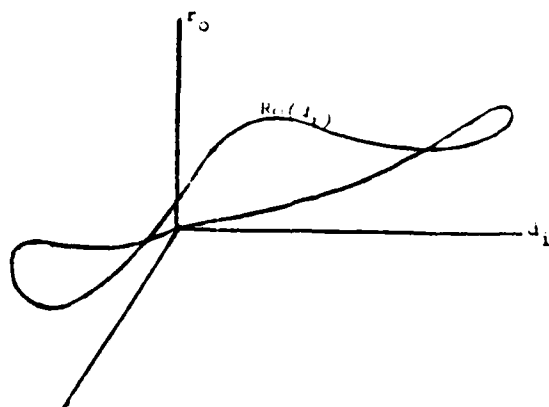


Fig 2a.

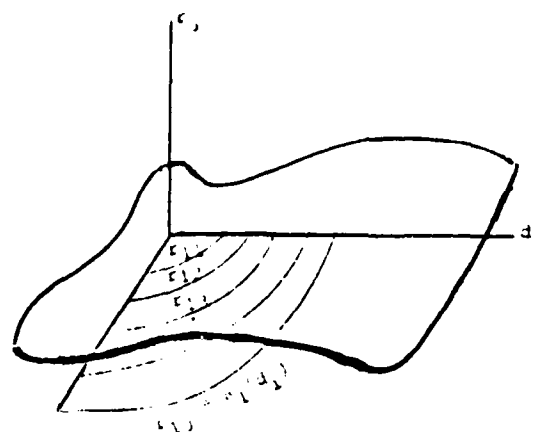


Fig 2b.

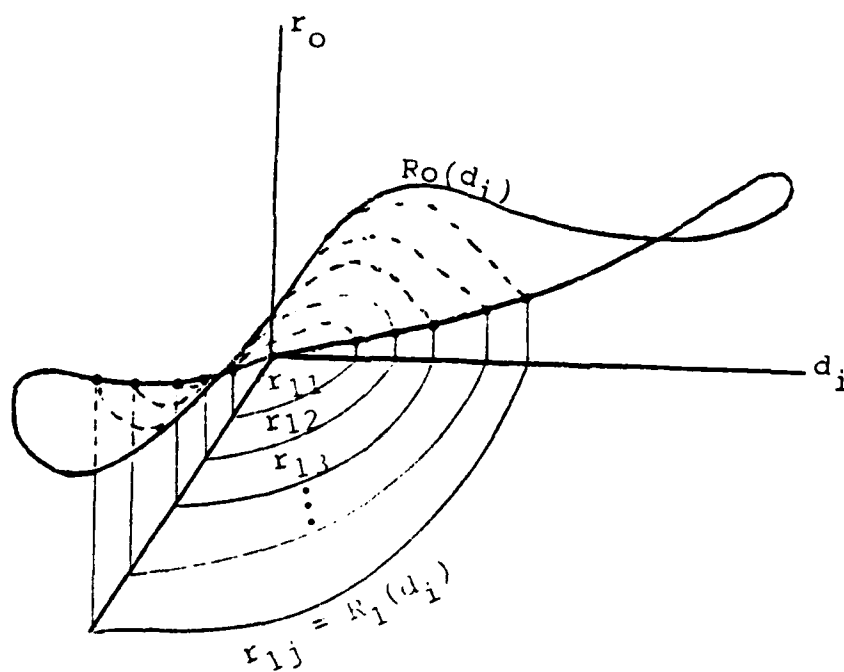


Fig 2c.

Fig 2. Response Surfaces' Constrained Maxima

$n+1$ problem dimension and collapsed into contour lines in the n dimensional domain. Thus, each constraint contour is a subset of the total surface $R_1(d_i)$ with a constant value r_{1j} . Therefore, the surface $R_1(d_i)$ is represented as a set of contour lines in the operating domain. Eq (10) depicts this set of j_{r_1} constraint contours. The number of contours required to characterize the surface is dependent on the surface complexity.

$$r_{1j} = R_1(d_i) \quad (10)$$

The question of contour interval criteria is determined in step 9 of the Methodology presentation. Figures 2a and 2b pictorially define the two measures of merit (MOE) which are to be optimized one to the other. Figure 2c combines Figure 2a's surface, $R_0(d_i)$, and Figure 2b's contours, $r_{1j} = R_1(d_i)$. The final step is to find a maximum value of $R_0(d_i)$ when constrained to each contour, $r_{1j} = R_1(d_i)$. Thus, there are j sets of Lagrange multiplier constrained maxima problems to be solved, Eq (11).

$$\begin{aligned} \text{optimize each } r_{0j} = R'_0(d_i, \lambda_j) = R_0(d_i) + \lambda_j \cdot (R_1(d_i) \\ - r_{1j}) \end{aligned} \quad (11)$$

The solution set (r_{0j}, r_{1j}) depicts the optimal $R_0(d_i)$ for a given $R_1(d_i)$ value over the entire operating domain (d_i) .

This theoretical presentation must be further constrained

for most applications. The theory allows the range of each domain element, d_i , to be unlimited. In practical application, the domain is limited to some experimental/operating range. In fact, it is this range which defines the research design and, therefore, limits the area to which the regression polynomial may be applied (6:336). This problem is exemplified in the case of the polynomials approximating each surface being second order. In this case, the Lagrange optimization will seek to reach the global critical point. This point, although mathematically correct, may lie outside the design region under consideration. Therefore, the upper and lower bounds for each domain element must be included in the problem, Eq (12). These additional constraints require a Lagrange multiplier for each bound.

$$u_i \leq d_i \leq l_i \quad (12)$$

Thus, for n domain elements, an additional $2 \cdot n$ λ 's must be included in the optimization function, $R'_0(d_i, \lambda_j, \lambda_{u_i}, \lambda_{l_i})$.

The methodology to solve this new inequality constrained optimization function is shown in Hadley & Whitin's text (13:436). Essentially, the process involves a pairwise comparison between inequality constraints and the original problem to determine which constraints are active. An active constraint is one which actively restricts the optimal domain of the original problem. These active constraints are set to equalities and the problem is resolved. The inactive

constraints are less restrictive than the active ones and therefore can be ignored. "The effort required to solve the problem increases rapidly with the number of [inequality] constraints," (13:437). Due to this increased effort, an alternative approach is also offered in step 10 of the methodology.

In conclusion, this section has shown the theoretical basis for the proposed methodology. Figure 2 has shown the two MOE's (r_0, r_1) in question as response surfaces (R_0, R_1) and collapsed the constraining surface into iso-value constraint lines ($r_{ij} = R_1(d_i)$) in the operating domain. Finally, the constrained maxima of the optimization surface subject to each contour is found using Lagrange multiplier techniques. It is the solution set of j maximized, ordered pairs which depict the optimal trade-off between the two surfaces. The remainder of this chapter is dedicated to the actual step-by-step process to achieve this optimization.

Stepwise Methodology

Figure 3 is a flowchart of the proposed methodology. It serves as an outline to this chapter and the entire study. The first three steps are systems analysis techniques and the remaining steps are optimization specific. Obviously, these first three steps cannot summarize the many systems analysis texts. However, this study is primarily concerned with an optimization technique; these three steps are in-

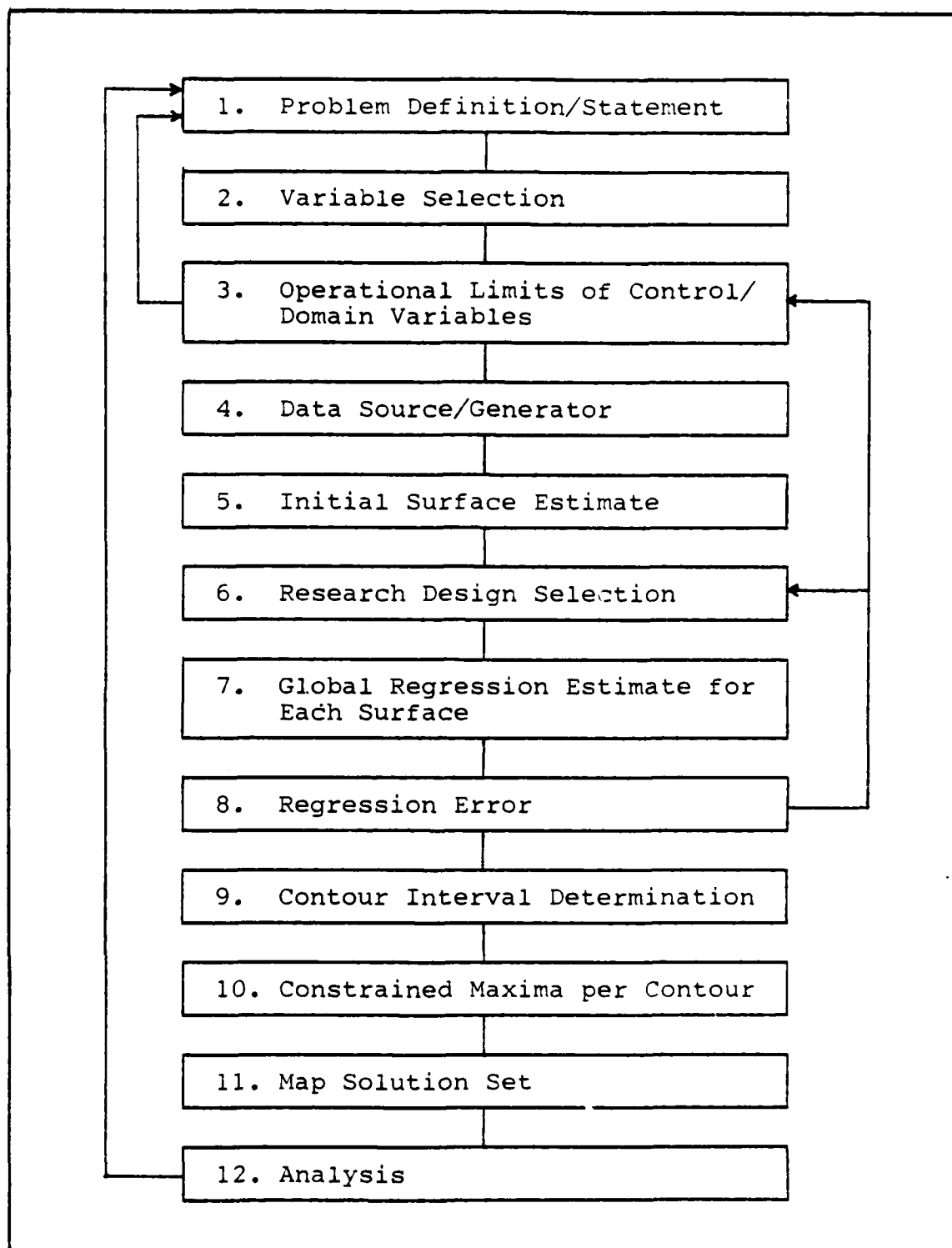


Fig 3. Methodology Flow Chart

cluded to re-emphasize their importance in a complete research project.

Step 1 - Problem Definition. The problem statement acts as the keystone and focus for the research. In defining the research problem, Quade (28:69) offers a series of questions to help the analyst interrogate the system's designers/decision makers. The purpose is to construct as lucid and specific a task definition as possible. One question bears reiteration throughout the study:

"Is it the right problem anyway? Might it not be just a manifestation or a symptom of a much larger or deeper problem? Would it be better to tackle this larger problem if there is one?" (Quade 28:69)

As the research progresses, this question must be reiterated since greater problem insight may require a redefinition of the research task. It is this iterative process which the flowchart, Figure 3, depicts with arrows returning to step one.

For Chapter Three's example, the problem is to find the optimal mix of a given set of strategic force types to fulfill two missions. The first mission is to maximize the counterforce (CF) capability given the force must meet the countervalue (CV) requirements. The second mission reverses the roles of the first and considers the maximization of CV given the required level of CF.

Step 2 - Variable Selection. The first step focuses the research on either the system's inability to reach an

aspiration level (goal) or a new goal of that system. Thus, step two begins by determining the system's objectives and their respective goals/aspiration levels set by the decision maker. Each goal's level of achievement is a measure of effectiveness and becomes the problem's dependent variable. The independent/control variables are defined by the domain in which the system is operating. One of the major tasks of the analyst will be to determine which of the domain elements have a dynamic effect upon the measure of effectiveness under consideration. Those that are correlated to the measures of effectiveness will become the control variables, while those that are not will be treated as constants. If, later in the research, data does not appear to be reacting properly, it may be caused by an actual control variable that was thought to be a constant and left out of the domain. An alternative perspective is to redefine the total system in terms of the model being assumed. This model consists of strictly the MOE's under consideration and those factors believed to be the primary cause of MOE changes. Thus, all other elements of the domain now become part of the environment in which this model operates.

The next section discusses the problems associated with scenario dependence in the selection of variables and their respective ranges.

Chapter Three's measures of effectiveness are levels of counterforce damage and countervalue damage as determined

by the aggregate arsenal exchange model. The control variables are the amount of each type of weapon available. For this example, four out of five weapons are ranged and one weapon is held constant. The number of variables not included as control variables for this study is large when considering the variables of the linear programming model, each of the right-hand-side values, and each of the equations in the matrix. Those variables not included in the study are discussed more thoroughly in steps four and five.

Step 3 - Operational Limits of Variables. This step defines the range of each control variable. Step two's variable selection combined with these ranges define the system for the research study and also the environment or scenario under which it will operate. Quade (28:193) discusses the importance of this scenario/environment to the overall analysis. He discusses how an analysis may be biased by the choice of scenario. Likewise, a well chosen scenario may aid in the robustness of the study to changes from the initial problem statement in step one. Therefore, the research should iterate back to step one to insure that the problem definition and research objective can be defined using the choice of variables in step two and the range of the control variables in step three.

For the example in Chapter Three, the range of the control variables is first selected to start at zero and run to the upper limits of 450, 750, 1040, and 424 for each

of the four weapons systems respectively. As we will see from the example, the lower limits of this problem were changed to 300, 450, 720, and 200 respectively, after initial analysis.

Step 4 - Data Source/Generator. In any research, the question of what resources will be available to the analyst must be addressed early in the study. These resources will include the personnel available, computer-time, the experimental or previous studies' data, and existing model availability. It will be from these resources that the data for analysis will be generated. The choice between resources will be determined by both the cost and time limitation upon the study and also the level of detail and variance acceptable for the solution.

For this research, the model is a deterministic, aggregated arsenal exchange model. The model uses linear programming to solve for the optimum use of weapons using damage expectancy. The model considers ten types of targets and five types of weapons. The objective function seeks to maximize the damage expectancy given the effectiveness of each type of weapon on each type of target and the capability to strike a target twice with the same weapon type. Further details of the model used to generate the data are available in Appendix A.

Since a simplex algorithm is generating each data point, the response surface is actually a surface of all

optimal points. Thus, for each n-tuple of the domain variables, there is a wide range of possible damage expectancy which could be considered, but the response surface only contains the optimal DE from each n-tuple of weapons.

As previously mentioned, the choice of control variable (weapons availability) is not inclusive of all possible variables in the linear programming (LP) algorithm. Therefore, the true functional relationship between the damage expectancy and the domain is not known due to the complexity of the LP algorithm. The LP domain consists of the study's control variables, the other right-hand-side values, all tableau variables, the number and type of equations in the matrix, and finally the objective function. Chapter One, Eq (1), depicts this actual functional relationship of the response over its LP domain. This actual relationship is unknown to the researcher and, therefore, the assumed regression model is chosen to estimate the relationship. In the next chapter's research example, only the control variables (right-hand-side weapon availability) are varied. Thus, the example regression approximates the true functional relationship of optimal DE with only the four weapons availability variables. This is, however, a good estimation of the surface, since changes across it will be a function of only those variables which are changing in the overall LP algorithm. In this case, the four out of

five weapons chosen for the study are the control variables. The effect of the constant fifth weapon is displayed in the B_0 coefficient of the regression polynomial.

Thus, a no-variance surface of optimal DE points is generated from the LP arsenal model and estimated in the four-variable regression model.

Step 5 - Initial Surface Estimation. The researcher's initial estimate for the regression model of the response surface should be determined by some prior knowledge of the actual system or a similar system. If the system's change in response is unknown, then either a simple, first order model, which may have a poor fit, is chosen, or a higher order model, which may require unnecessary sampling, is chosen.

For this study, prior knowledge of similar systems was available from Smith and Mellichamp's work (30). They found, in studying a similar model, that a second order regression polynomial gave an excellent prediction of the damage expectancy caused in a five-weapon arsenal. Based on this prior knowledge, the study estimates the example arsenal will respond in a similar manner and the damage expectancy will be a monotonically increasing function which may require a second order regression polynomial.

Step 6 - Research Design Selection. Chapter One displays the power of the research design as a sampling tool to gather data points for use in the regression

estimate. This step now defines the criteria to choose a design. Smith and Mellichamp (30) displayed the analytic advantage of independent co-efficients for the comparative trade-off between control variables with respect to the MOE. Therefore, one of the selection criteria will be orthogonality.

A further criterion is brought about by the use of a deterministic model. As shown in Chapter One, the deterministic model has no measurement variance and, thus, a primary criterion now becomes that of minimizing the bias. Unfortunately, Karson's minimum bias criteria is only shown for a two-level design (17) and no minimum bias, only 3^k designs, exist. Hence, from the previous step we will need a design which will allow for a second order equation and orthogonality.

This criterion is satisfied by Box and Behnken's fractional factorial 3^k design (2). This 3^k design samples points from each corner of a cube and each point of a star which is co-located with the surface of the cube and at the center of the cube. Appendix B shows this research design and the value of each of the measures of effectiveness for each sample point. The design is given in two formats, coded and uncoded. The coded format has transformed the range of each domain variable from upper and lower limits to plus and minus one respectively, and zero for the mid-point. Eq (13) shows this transformation where \bar{x} is the mid-point in the range between the upper and lower limit

and d is the actual range. Thus, for the example in Chapter Three, the first domain variable, w_1 , ranges from 300 to 450. For this case, \bar{x} is 375 and d is 150.

$$\text{coded } x = (x - \bar{x})/d/2 \quad (13)$$

Using Eq (13), we see that the lower limit of 300 is transformed to a -1 and the upper limit of 450 to +1 and the midpoint to zero. The transformed design simplifies checking for the orthogonality of the research design. Likewise, if said regression algorithm does not have an automatic covariance computing technique, the transformed matrix may be used.

For example, in Chapter Three, the power of this design can be displayed in the number of samples which will be required for the combinations of the ranges of each of the domain variables. If a full enumeration of the surface were done, it would require greater than three billion samples. However, the example research design uses 25 points. It should be noted that the original Box and Behnken design required an additional two repetitions on the center point for degrees of freedom to estimate the variance error. However, this repetition of the center point is not required since this example uses a deterministic model with only bias error.

Step 7 - Global Regression Estimate. Eq (1) shows the functional relationship between the measure of effectiveness

and its control variables. This step now estimates that functional relationship in a regression polynomial using the sampled points from the design in step six. There are many computer regression packages available to do this. For the case in point, the Statistical Package for the Social Sciences (SPSS) (25:320) is used to calculate the regression equation. A stepwise regression option is chosen, thus, allowing the researcher to follow the introduction of each independent coefficient and re-check for this independence. A sample of the regression output is found in Appendix C. It should be noted that since we are using a deterministic model, the F statistic is meaningless except as a ratio of the sum of squares explained to the sum of squares unexplained, which may be an indicator of the bias error.

The normal statistical inferences drawn from the F statistic cannot be used in this deterministic regression model. The reason lies in the basic assumption that there is a distribution of y for each value of x in the regression (33:15). Thus, the assumption can be thought of as a conditional probability, $P(Y_i | x_i)$. However, with the use of a deterministic model, this conditional probability equals one. Therefore, the error term in our generalized regression equation is not a measure of both the measurement variance and the model bias but, rather, only the bias due to the assumed model and its inability to fit the actual

response surface. Since the F statistic is a ratio of the variance explained by the regression to the unexplained variance, we see that with the use of this deterministic model, the unexplained variance is actually a measure of the lack of fit of our regression polynomial to the surface divided by the degrees of freedom, Eq (14). Since the model has no variance in measurement, it would be wrong to make statistical inferences as to the sample population from the F statistic.

$$F = \frac{\text{Var, Explained}}{\text{Var, Unexplained}} = \frac{(\hat{y}_i - \bar{y})^2/p}{(\hat{y}_i - y_i)^2/(n-p)} \quad (14)$$

where

p = # of regression variables

Step 8 - Regression Error. This step tests the regression polynomial's goodness of fit for each surface. As stated earlier, the normal F statistical inferences cannot be used. Therefore, the primary criteria will be residual error between the predicted and actual values. The residual error predicted for each of the design points is included in the SPSS output. To increase the confidence in the regression equation, a random sampling of residual error is also taken.

Step 9 - Contour Interval Determination. As mentioned in Chapter One, we must represent our constraining surface as a set of iso-value contour lines. The criteria as to how many lines or the interval between these lines lies pri-

marily in the last two steps. If the error is small in our prediction from our regression polynomial, then the interval between our contour lines may, likewise, be small. The purpose of the contour is to show the change in slope of the surface. Thus, if we are unable to distinguish a change in slope, there is no point in having a contour at that point in the projection of the surface. Using this logic, we can look at the standard deviation of the error from steps seven and eight above. If we assume that we will be unable to distinguish a change in response unless it is outside of the error of the estimate, then we would not want a contour interval any closer than that of the error. Thus, an interval criteria can be established based on the standard deviation of residual error. Since the regression has only bias error of a least square fit of the design data, the bias should be a normal distribution. Using this assumption, a range of two standard deviations of residual error would be a minimum interval for selection. Outside this range, one would be assured that at least 97% of the time, the model will be able to estimate a change in the response surface that was not caused by the regression error. This criteria then sets a minimum value for the contour interval. The maximum interval must be determined by the complexity of the surface and the degree of accuracy required in the solution.

For the example in Chapter Three, the standard devia-

tional errors were so small that the interval was arbitrarily chosen at a value larger than the minimum that could be allowed. This arbitrary choice was based on an estimate believed to be a sufficient number of samples to adequately define the trade-off between the two measures of effectiveness.

Step 10 - Constrained Maxima Determination. As described earlier in the chapter, Lagrange multipliers can now determine the maximum of response R_0 , as compared to each contour line found in the preceding step. However, it was also shown that the Lagrange multiplier technique could become tedious with the number of multipliers required for the inequality constraints. Thus, with the number of constraints and no computer libraries readily available to solve the Lagrange technique, an alternative optimization technique was found. This technique is based on the work of Fiacco and McCormick (12) in nonlinear programming. The actual computer library that is used was developed by Mylander (23). This algorithm finds the minimum of either a linear or nonlinear, multivariate objective function, subject to linear or nonlinear equality and inequality constraints. The algorithm uses Fiacco and McCormick's modified objective function which is formulated by using the original objective function and a penalty function. It then starts an iterative process in choosing new estimates to minimize this new objective function. The

routine estimates each new iterative answer by an extrapolation process. It then tests for convergence. In seeking its next iterative solution, it also uses a gradient search routine to speed the process. This is a powerful algorithm which solves Fiacco and McCormick's function and also simultaneously solves a dual problem based on Lagrange's technique. For further discussion of the theory, see Chapter 8 of Fiacco and McCormick (12) and also Appendix D, a description of the SUMT library as composed by Captain R. M. Floyd for the Air Force Institute of Technology.

Step 11 - Map Solution Set. The solution set consists of the maximum points for each constraint contour, $(\max r_0, r_1)_j$. The trade-off between the two measures of effectiveness may be displayed graphically by plotting the solution pairs. The slope of this graph yields the trade-off of marginal return for the maximum of r_0 given a value of r_1 . Likewise, the n-tuples which created this optimal set and which also come from the SUMT program can be plotted, allowing a visual picture of the correlation between each domain variable and its use in this trade-off.

Step 12 - Analysis. This final step is the interpretation of the data that is gathered in the past eleven steps. The data mapping of step eleven, when compared to the independent regression coefficients of each weapon system, will give insight into the effectiveness of each control

variable upon the single measure of merit, the multiple measures of merit, and the optimal trade-off in both. Chapter Four of this study will show the comparisons and insights through this technique.

CHAPTER THREE

Application Example

Introduction

This chapter lends numeric reality to Chapter Two's proposed methodology. The application is a generic military force structure problem similar to that studied by P. W. Smith and J. Mellichamp (30). All but the last two steps of the methodology will be addressed in this chapter. The data mapping and analysis are postponed until Chapter Four. The chapter presentation closely follows the actual chronological order of the research.

As stated earlier, this generic force structure problem is similar to that studied by Smith and Mellichamp. Their research shows the individual effectiveness of each weapon system when used in concert with the remaining arsenal available. Their study uses a single measure of effectiveness (MOE), damage expectancy (DE). Their use of independent regression coefficients shows both the incremental MOE value of each weapon and the marginal return of one weapon to another. Building on Smith and Mellichamp's work, this study considers a similar force and then compares the optimal trade-off between two missions (MOE's), counterforce (CF) and countervalue (CV). The optimization of this trade-off is shown in the following methodology application.

Problem Definition

The decision maker must determine the best mix of forces to fulfill two missions. The first is a counter-force (CF) mission and the second is a countervalue (CV) mission. The force structure must be optimized so that it has the flexibility to perform one mission or the other. The alternatives for the structure are the possible combinations of five different weapon systems' respective ranges of values. The fifth weapon system is set at a fixed level of 360 warheads prior to this analysis. The decision maker's problem is to determine the optimal force structure for a range of different CV demands (mission requirements) and for the remaining four weapon systems.

The generic weapon characteristics are shown in Table I. The weapons (w_i) may be thought of as representative components of an arsenal. For instance, w_1 and w_2 may be thought of as two different types of ICBM's and w_3 may be an SLBM component. For this problem, w_5 is assumed to be introduced at a constant level of 360 warheads. The two characteristics, yield and circular error probable (CEP), define the level of destructive force on a target. CEP is the radius within which the guidance of the system will land the warhead fifty percent of the time. CEP and weapon yield are given in thousands of feet and megatons respectively.

TABLE I
Weapon System Characteristics

	W ₁	W ₂	W ₃	W ₄	W ₅
CEP	.200	.150	.240	.600	.200
Warheads/Weapon	1	3	10	4	6
Yield	2.0	.17	.05	.35	.20
Reliability	.85	.85	.85	.85	.85
Daily Alert	.85	.85	.85	.33	.33
Max Warheads/Type	450	750	1040	424	360

The target base defines the remainder of the system. Table II shows the characteristics of the targets. There are six targets in the CF class and four in the CV class. The CF type targets may be representative of such things as ICBM silos, military bases, and C³I facilities. Likewise, the CV type targets may be representative of such things as storage or industrial facilities. The vulnerability number in Table II is the connection between the target and the weapon.

The vulnerability number (VNTK) consists of five alpha numeric characters (1:34). The first two numeric characters represent the target's level of hardness to its most vulnerable kill mechanism. The alpha character identifies the kill mechanism most effective against that target.

TABLE II
Target Base Characteristics

Type	Number	Vulnerability Number	Diameter
CV1	140	24Q0	.56
CV2	215	13P1	.49
CV3	430	31P6	.00
CV4	520	23Q0	.31
CF1	450	35P7	.00
CF2	1000	52P8	.00
CF3	200	39P0	.00
CF4	20	22P1	.36
CF5	150	11P0	.00
CF6	100	10Q1	.79

Finally, the last number shows the target's susceptibility to increased duration of the particular effect. The linear programming arsenal exchange model requires these VN_{TK}'s to be converted into single-shot probability of kill (SSP_k) coefficients for the objective functions. The arsenal exchange work of Robert Bunnell and Richard Takacs (5) required the computer coding of this conversion from VN_{TK} to SSP_k . With their kind help, this program has been employed to compute the SSP_k 's for this study. Further explanation of the

target characteristics and weapons characteristics used in this aggregated arsenal exchange model are given in Appendix A.

Variable Selection

As mentioned in the previous chapter, there are many possible variables in the arsenal exchange linear programming model. For this study, we are primarily concerned with the objective function value and the right-hand-side value for the number of weapons available. The ability to destroy the target base in question is measured by the objective function value. Since two separate missions are considered, CF and CV, the measure of effectiveness is the level of destruction to each of the respective target bases. This study will refer to each MOE as CF or CV respectively, where each is a measure of the objective function for that particular model. Likewise, for consideration of a combined mission of both CF and CV, this study will refer to this as all forces (AF).

The control variables will be the five weapon systems which compose the proposed arsenal. Of these five systems, the first four will be varied in the study and, therefore, included in the regression analysis. The fifth system is constant and will be shown in the regression section to be in the B_0 term of the polynomial. Similarly, all the other variables in the LP which remain constant do not have to be included in the control variables. This selection of control variables thus assumes that the target base will

remain constant, otherwise the change in target base would likewise have to be considered in the regression analysis and also in the sampling from the LP model. This variable selection, then, leads to the four weapons as control variables and the two measures of merit, CF and CV.

Control Variable Limits

The range of each control variable is determined by the scenario or the study under consideration. For this example, weapon five is introduced and maintained at a constant level of 60 weapons or 360 warheads. The remaining four weapons are assumed to have previously determined upper limits shown in Table I. Since all but one of the weapon systems has multiple warheads, the \dot{w}_i variables are in terms of warheads rather than actual weapon numbers. The lower bounds for these variables are dependent on the range/degree of conflict to be considered. Initially, the lower limits are set at zero. However, after the regression error was determined in step eight, the lower limits were re-evaluated and set as shown in Table III. This revision is further explained in step eight, Regression Error.

Data Source/Generation

The data for this generic problem are generated from the aggregated arsenal exchange model, which is detailed in Appendix A. The choice of this particular model was not

TABLE III
Control Variables' Ranges

Weapon Ranges (Warheads)	
Initial	Revised
$0 \leq W_1 \leq 450$	$300 \leq W_1 \leq 450$
$0 \leq W_2 \leq 750$	$450 \leq W_2 \leq 750$
$0 \leq W_3 \leq 1040$	$720 \leq W_3 \leq 1040$
$0 \leq W_4 \leq 424$	$200 \leq W_4 \leq 424$
$W_5 = 360$	$W_5 = 360$

critical to the methodology example; therefore, a simplified version was chosen.

The model attempts to maximize the DE from the weapons available on the given target data base. The separate missions are modeled by using either the CF or the CV components in the objective function. Thus, each mission has the same LP tableau except for the change in objective function. The two measures of effectiveness, CV and CF, are determined by the two different objective functions respectively. Since all other LP components remain constant, the changes in the respective MOE are due solely to changes in the right-hand-side control variables. Thus, the two MOE variables are dependent upon the combinations

of independent variables, w_i , levels. The function of the independent variables, w_i , is the simplex algorithm, the objective value of which is the MOE in question.

Initial Surface Estimation

As mentioned earlier, the initial surface estimation is based on the study of Smith and Mellichamp (30). In that study, they found the DE to be a monotonically increasing function of the force structure. Thus, the study assumes that the contour of each surface, CF and CV, will be similar to that of the combined mission which Smith and Mellichamp studied. In light of this, a second order regression model is assumed.

Research Design Selection

Chapter Two shows the criteria for selection of a research design. In particular, Smith and Mellichamp's analysis (30) relies on independent coefficients which require an orthogonal design. A minimum bias design is optimal for the deterministic model but, as previously mentioned, there are none available for greater than two dimensions. Thus, the criteria of rotatability, equispacing, and orthogonality were chosen to minimize the variance and bias. To fulfill these criteria, the Box and Behnken's fractional factorial 3^k design was chosen (2). The original design calls for 27 samples. This design was modified to 25 samples because the model is deterministic. The difference

TABLE IV
Research Design and Sampled Values

Design Points (Coded)				CF	CV	AF
450(1)	750(1)	880(0)	312(0)	932.67	1190.02	1736.46
450(1)	450(-1)	880(0)	312(0)	868.40	1159.59	1618.73
300(-1)	750(1)	880(0)	312(0)	862.12	1170.66	1652.86
300(-1)	450(-1)	880(0)	312(0)	797.85	1139.65	1504.04
375(0)	600(0)	1040(1)	424(1)	873.84	1165.56	1677.36
375(0)	600(0)	1040(1)	200(-1)	868.13	1165.56	1645.10
375(0)	600(0)	720(-1)	424(1)	865.41	1164.70	1626.09
375(0)	600(0)	720(-1)	200(-1)	859.61	1164.31	1585.47
450(1)	600(0)	880(0)	424(1)	904.92	1174.87	1701.47
450(1)	600(0)	880(0)	200(-1)	899.15	1174.81	1668.05
300(-1)	600(0)	880(0)	424(1)	834.40	1153.39	1602.27
300(-1)	600(0)	880(0)	200(-1)	828.60	1155.32	1567.22
375(0)	750(1)	1040(1)	312(0)	901.59	1180.83	1717.52
375(0)	750(1)	720(-1)	312(0)	893.14	1179.81	1673.61
375(0)	450(-1)	1040(1)	312(0)	837.39	1150.28	1594.91
375(0)	450(-1)	720(-1)	312(0)	828.87	1148.42	1526.83
450(1)	600(0)	1040(1)	312(0)	906.24	1175.30	1708.49
450(1)	600(0)	720(-1)	312(0)	897.79	1174.28	1657.15
300(-1)	600(0)	1040(1)	312(0)	835.76	1155.82	1612.06
300(-1)	600(0)	720(-1)	312(0)	827.24	1153.96	1554.73
375(0)	750(1)	880(0)	424(1)	900.26	1180.40	1710.86
375(0)	750(1)	880(0)	200(-1)	894.50	1180.20	1681.32
375(0)	450(-1)	880(0)	424(1)	836.03	1149.85	1585.12
375(0)	450(-1)	880(0)	200(-1)	830.23	1149.79	1540.05
375(0)	600(0)	880(0)	312(0)	877.77	1165.13	1635.88

is in the repetition of the center point which is required for degrees of freedom in an experiment which has no variance. Table IV displays this design for the revised limits in both its coded and uncoded version, as described in Chapter Two. The coded values are shown in parentheses next to their uncoded counterparts. This table also shows the sample values of each MOE (CV, CF, AF) for these respective design points. A further description of the design is in Appendix B.

Global Regression Estimate

The above-mentioned design yields 25 data points for use in a regression analysis. Each regression looks at the MOE regressed on the four control variables, the cross products of these variables and the square terms of these variables, Eq (15).

$$\begin{aligned}
 \text{MOE } (w_i) = & B_0 + B_1W_1 + B_2W_2 + B_3W_3 + B_4W_4 + B_{11}W_1^2 \\
 & + B_{22}W_2^2 + B_{33}W_3^2 + B_{44}W_4^2 + B_{12}W_1 \cdot W_2 \\
 & + B_{13}W_1 \cdot W_3 + B_{14}W_1 \cdot W_4 + B_{23}W_2 \cdot W_3 \\
 & + B_{24}W_2 \cdot W_4 + B_{34}W_3 \cdot W_4
 \end{aligned} \tag{15}$$

The computer regression library chosen was the Statistical Package for the Social Sciences (SPSS) with a stepwise option (25:330). An example of a full regression is shown in Appendix C. Likewise, the final steps of each of the other regressions is also given in this Appendix.

The choice of the stepwise option allows the researcher to follow the introduction of each coefficient and display its independence from the other coefficients. Eqs (16), (17), and (18) are the respective regression polynomials using the initial limits shown in Table III. When applying the revised limits' design points, the regression polynomials are found to be Eqs (19), (20), and (21) respectively.

$$\begin{aligned}
 AF = & 302.022 + 0.91918W_1 + 0.94721W_2 + 0.87583W_3 \\
 & + 0.82253W_4 - 0.00008W_1^2 - 0.00014W_2^2 - 0.00027W_3 \\
 & - 0.00018W_4^2 - 0.00011W_1W_2 - 0.00002W_1W_3 - 0.00002W_1W_4 \\
 & - 0.00019W_2W_3 - 0.00022W_2W_4 - 0.00049W_3W_4
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 CF = & 370.603 + 0.7024W_1 + 0.4473W_2 + 0.1813W_3 + 0.1604W_4 \\
 & - 0.00016W_1^2 - 0.000135W_2^2 - 0.00007W_3^2 - 0.00003W_4^2 \\
 & - 0.00020W_1W_2 - 0.00002W_1W_3 - 0.00004W_1W_4 - 0.00003W_2W_3 \\
 & - 0.00004W_2W_4 - 0.00013W_3W_4
 \end{aligned} \tag{17}$$

$$\begin{aligned}
 CV = & 279.659 + 1.2307W_1 + 1.0376W_2 + 0.5759W_3 + 0.7330W_4 \\
 & - 0.00035W_1^2 - 0.00035W_2^2 - 0.00016W_3^2 - 0.00036W_4^2 \\
 & - 0.00070W_1W_2 - 0.00046W_1W_3 - 0.00037W_1W_4 - 0.00036W_2W_3 \\
 & - 0.00026W_2W_4 - 0.00040W_3W_4
 \end{aligned} \tag{18}$$

$$\begin{aligned}
AF = & 323.747508 + 1.357119W_1 + 1.335273W_2 + 0.5225785W_3 \\
& + 0.4350739W_4 - 0.0002055W_1^2 - 0.0002804W_2^2 - 0.0000657W_3^2 \\
& - 0.0000239W_4^2 - 0.0006909W_1W_2 - 0.0001248W_1W_3 \\
& - 0.0000485W_1W_4 - 0.0002518W_2W_3 - 0.0002311W_2W_4 \\
& - 0.0001166W_3W_4
\end{aligned} \tag{19}$$

$$\begin{aligned}
CF = & 504.542366 + 0.4715619W_1 + 0.2955107W_2 + 0.0288839W_3 \\
& + 0.0278609W_4 + 0.0000003W_1^2 - 0.0000671W_2^2 - 0.0000006W_3^2 \\
& - 0.0000005W_4^2 - 0.0000015W_1W_3 - 0.0000009W_1W_4 \\
& - 0.0000007W_2W_3 - 0.0000006W_2W_4 - 0.0000013W_3W_4
\end{aligned} \tag{20}$$

$$\begin{aligned}
CV = & 1027.039 + 0.1658222W_1 + 0.1198222W_2 + 0.0307574W_3 \\
& + 0.0019064W_4 + 0.0000054W_4^2 - 0.0000153W_1^2 - 0.0000040W_2^2 \\
& - 0.0000075W_3^2 - 0.0000129W_1W_2 - 0.0000175W_1W_3 \\
& - 0.0000087W_2W_3 - 0.0000054W_3W_4
\end{aligned} \tag{21}$$

The first SPSS regression of the revised limits rounded the coefficients to the fourth decimal place. Thus, it dropped most of the second order terms from the CF equation and all of the second order and cross terms from the CV equation. A second regression on the revised samples was run to allow the seven decimal places in Eqs (19), (20), and (21). These small coefficients are probably due to the smaller region and slow change of surface contour in this region. The next section considers the goodness of fit of

these polynomials to the actual surfaces and explains the reason for revised area of consideration.

Regression Error

At this point in the research, it was determined that the initial regression polynomials lacked the degree of accuracy sought in the solution. This can be seen by looking at the standard deviation of the residual error of each regression. The standard deviations were 17.4, 7.9, and 20.3 for all force, counterforce, and countervalue respectively. Since this error was thought to be unacceptable, further error random sampling was not accomplished.

In accordance with the methodology, the study returned to steps six and three of the methodology. In step six, Research Design Selection, the Box and Behnken design was reviewed for its orthogonality and all data points were rechecked as to accuracy. This review showed the area of consideration to be too large for the second order response surface. Thus, a redefinition of the problem scenario or level of conflict was considered. The redefinition found the more realistic ranging of variables listed in Table IV. Therefore, the revised lower limits were applied in step six, a new sampling taken, and the methodology proceeded as designed.

The error for the second set of regression equations, Eqs (19), (20), and (21), was found to be much smaller. Table V summarizes the regression error for the CF, CV, and

AF surfaces respectively. The table shows the high degree of accuracy with the revised control variable limits. The CF and CV estimates are accurate to within a single unit for even three standard deviations of error. This small error is significant in the next section.

TABLE V
Regression Error Summary

Surface	Design S.D. Of Error	Random Mean Error	Random S.D. of Error
CF	0.0131	0.223	0.204
CV	0.1569	0.234	0.222
AF	1.0066	1.233	1.154

Contour Interval Determination

Chapter Two determined the criteria for minimum interval to be no closer than two standard deviations of error. From the above-mentioned regression error, the minimum interval for counterforce and countervalue constrained surfaces was determined to be no less than one unit. In the case of AF, it was determined that it would be no less than three units. In actual fact, it was not necessary to utilize such small steps to show the optimal trade-off between surfaces. A step of two units was more than sufficient for counterforce and countervalue and five units for all force.

Constrained Maxima Determination

As described in the previous chapter, the SUMT program was used to optimize and find the maximum of one surface constrained to the contours of another. For the planned analysis in the next chapter, three cases of optimization are looked at. The first two cases consider the optimization of CF given a value of CV and CV given a value of CF. Both these cases use simplified, regression polynomials described in the next chapter. The third case compares the difference between the use of simplified and detailed regression polynomials. The data for these cases are contained in Appendix D.

Conclusion

This chapter has applied the proposed methodology to a generic, military force structure problem. The final results are compiled in Appendices D and E. The next chapter provides an analysis of these results.

CHAPTER FOUR

Analysis

Introduction

This chapter presents the study's analysis utilizing Chapter Three's example data. The analysis goals are twofold. One is to develop insight into the optimization process between the measures of effectiveness (MOE's) and their respective domain elements (n-tuples) in achieving the constrained maxima. The second goal is to offer the decision maker this analysis in a usable format. Prior to the analysis, redefinition of Chapter Three's works and some simplifying assumptions are made.

In Chapter Three, the MOE response surface is generated from a linear programming optimization model. Thus, each point on the surface represents the optimal use of its respective domain elements (n-tuple). Since each point is an optimal, the surface displays the optimal MOE for the system's entire range of domain elements. Therefore, the regression polynomials in Chapter Three (Eqs (19), (20), and (21)) represent the regression estimates for the optimal employment of the domain elements to achieve their respective MOE. In terms of the example, each equation represents the optimal force structure to achieve its respective mission (MOE). Using the above-mentioned polynomial definitions, the SUMT program selects from a domain subset (force), already optimally designed to fulfill one

MOE (mission), the best mixture of domain elements (weapon systems, w_1) to fulfill a second MOE (mission). In the analysis, the first goal is to determine the cause for the optimal trade-off between these MOE's and the domain n-tuples which generate this trade-off. To aid this discussion of constrained maxima (optimal trade-off), the regression polynomials are simplified by rounding the coefficients to four decimals.

These simplified polynomials, Eqs (19a), (20a), and (21a), display three major points of discussion in the chapter.

$$\begin{aligned}
 AF = & 323.7475 + 1.3571*W_1 - 0.0002*W_1^2 - 0.0007*W_1W_2 \\
 & - 0.0001*W_1W_3 + 1.3353*W_2 - 0.0003*W_2^2 \\
 & - 0.0003*W_2W_3 - 0.0002*W_2W_4 + 0.5226*W_3 \\
 & - 0.0001*W_3^2 - 0.0001*W_3W_4 + 0.4351*W_4
 \end{aligned} \tag{19a}$$

$$\begin{aligned}
 CF = & 504.542 + 0.4716*W_1 + 0.2955*W_2 - 0.301*W_2^2 \\
 & + 0.0289*W_3 + 0.0279*W_4
 \end{aligned} \tag{20a}$$

$$\begin{aligned}
 CV = & 1027.039 + 0.1658*W_1 + 0.1198*W_2 + 0.0308*W_3 \\
 & + 0.0019*W_4
 \end{aligned} \tag{21a}$$

First, they are used to show a graphic representation of the optimization process of an MOE surface constrained to a single contour; thus, giving some pictorial validity

by demonstrating graphically the results of SUMT. Second, the simplified regression equations aid in the analysis of the MOE's optimal trade-off and the priority process for entering domain variables. Finally, it is hoped that while these equations serve to improve the quality of discussion, they also serve to exemplify possible pitfalls in rounding the regression coefficients.

The chapter presentation begins with the above-mentioned graphic representation of the optimization process. This is accomplished through a series of MOE contour plots, one type MOE overlaid upon another type MOE. Following the graphic optimization, the data from Chapter Three is mapped and analyzed over the range of domain elements. The chapter concludes with a summary of the methodology's advantages to the decision maker.

Graphic Optimization

This section graphically displays the SUMT optimization of CF for a given CV contour. The simplified regression polynomials, Eqs (20a) and (21a), are used in this section of the analysis.

In Chapter Three, CF was optimized for CV values of 1154 to 1224 in two unit steps. These CV lower and upper limits are determined by the lower and upper limits of the domain elements. The standard deviation of error for CV was found to be 0.222. Thus, the minimal interval criteria was evaluated to be one unit. This close an interval was not

necessary to adequately represent the trade-off between CF and CV. Thus, the contour interval of two units was chosen. Appendix E contains a full listing of the example MOE comparisons and their respective domain elements. A review of these data shows W_2 is constant at 450 in the region of $CV = 1176$. Therefore, the graphic example uses $CV = 1176$ and $W_2 = 450$.

In this experimental region ($CV = 1176$, $W_2 = 450$), the contour plots consist of variables W_1 , W_3 , W_4 , and the MOE. This requires representation of four dimensions on a two-dimensional plot. This is accomplished through multiple contours.

The comparison data plots for CF and CV are generated by a computer program which iterates each feasible combination of control variables for each regression polynomial. These data were then plotted and are available in Appendix E. Several of the plots in Appendix E have been reduced in scale and are used in combined overlays throughout the remainder of the chapter.

Figure 4 represents the counterforce contour value of 850 and also 860 (dashed line) with ordinate W_4 and abscissa W_3 . Each diagonal line across the graph is a contour of both CF and weapon one. As such, each is identified by the pair of numbers in parentheses. The first is the MOE and the second is the W_1 value.

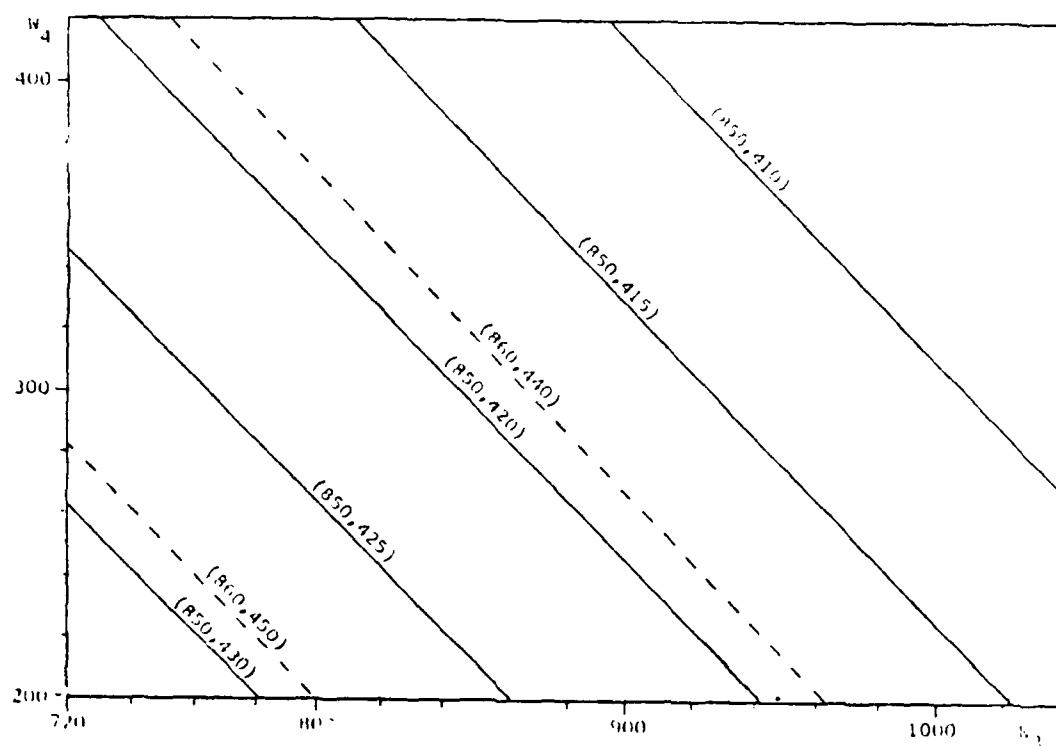


Fig 4. Contour Plot of $CF = 850/860$ and $W_2 = 450$

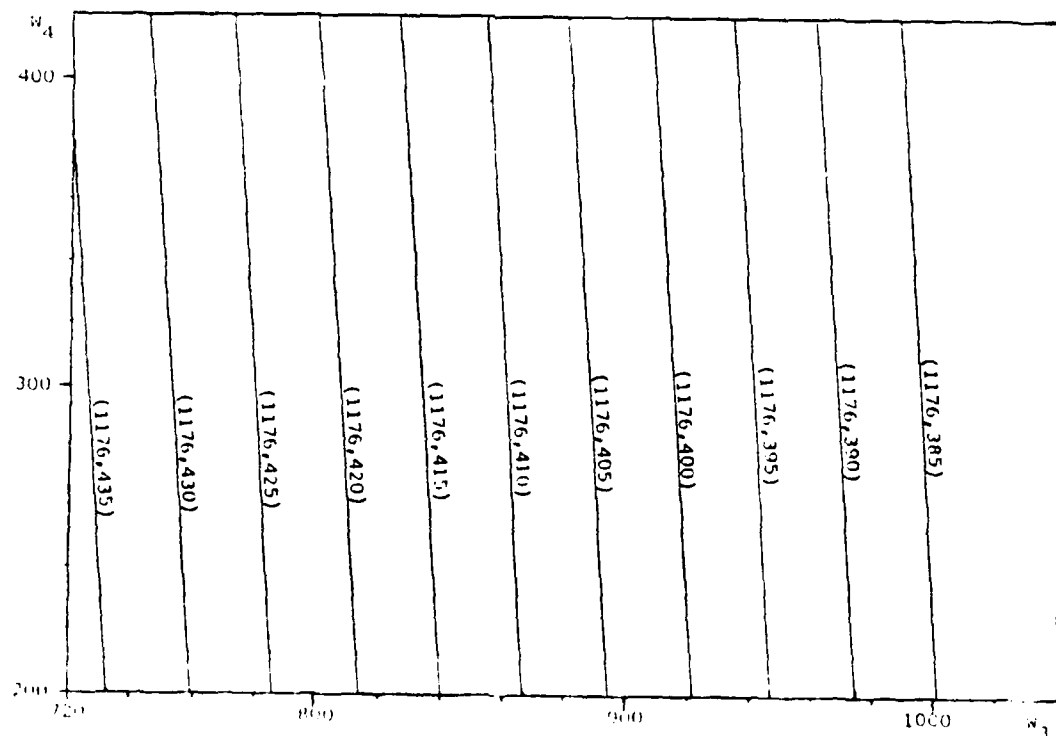


Fig 5. Contour Plot of $CV = 1176.0$ and $W_2 = 450$

Figure 5 represents the countervalue of 1176 with the various diagonal lines due to the stepwise increase in W_1 . This representation of five-space in two-dimensional plots is similar to that used by Smith and Mellichamp (30).

If Figures 4 and 5 are combined, the intersection of common W_1 value contours reveals the area of common domain between surfaces. Thus, the graphic comparison optimization becomes a search for the maximum CF value with a common W_1 value on both graphs. Figure 6 is the overlay of Figures 4 and 5.

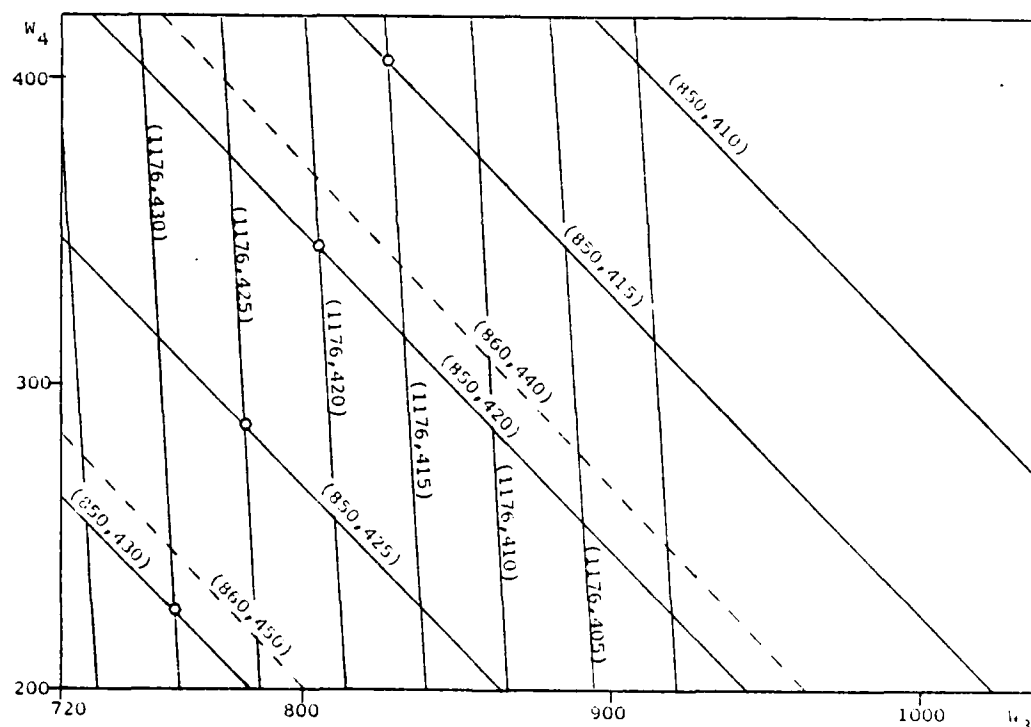


Fig 6. Overlay Plot of CV = 1176 and CF = 850, 860

The figure displays a commonality in the 1176 CV contour and the 850 CF contour; however, there is no common domain point(s) for the 860 CF contours and the contours of 1176 CV. Therefore, Figure 6 has bounded the graphic optimization search between CF values of 850 and 860.

Figures 7 and 8 have similarly overlaid the CV plot of 1176 against the CF plots of 854 and 856 respectively. Figure 7 shows the common points for values of W_1 at 435 and 430. Figure 8 has no common points. However, Figure 8 does show the closest point to a common W_1 value of 435. This point is just off the W_4 axis and is marked "Z."

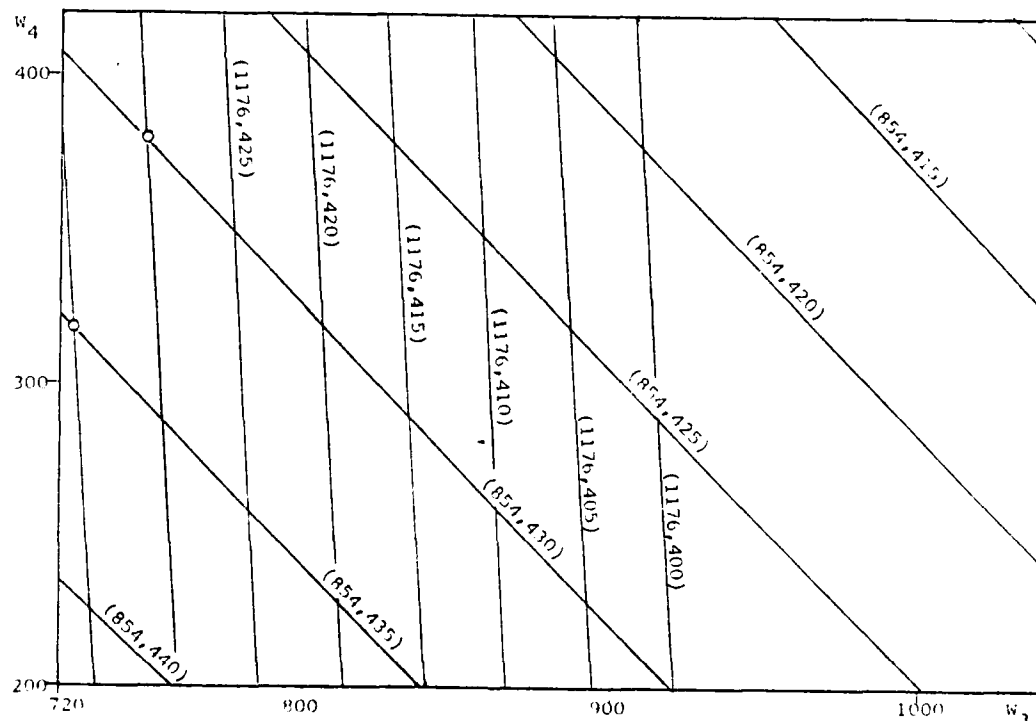


Fig 7. Overlay Plot of CV = 1176 and CF = 854

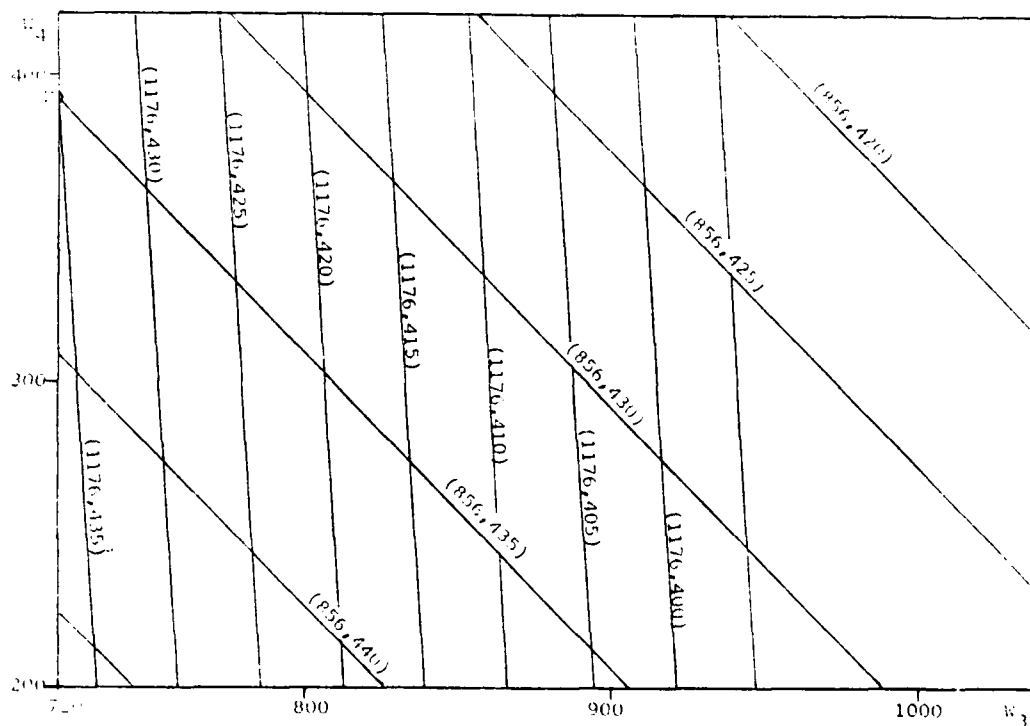


Fig 8. Overlay Plot of $CV = 1176$ and $CF = 856$

Graphically, the optimal point lies between the CF values of 854 and 856, and the W_1 value lies between 430 and 435. This graphic solution compares very well with that predicted by the SUMT optimization in Appendix E. The SUMT operating domain is 434.9, 450, 720, and 400.3 for W_1 through W_4 respectively. The graphic solutions between Figures 7 and 8 show this to be the operating domain upon which the plots converge. Likewise, SUMT's predicted maximum CF of 855.36 very closely approximates the "Z" point just off the axis of the CF 856 plot.

Therefore, the overlay contour plots in Appendix E have graphically validated the optimization process used in the SUMT algorithm. This plot overlay technique will be applied again in the next sections.

Data Presentation

At the conclusion of Chapter Three, the data had yet to be presented. This section displays the data mapping that will be used in the following section for the analysis. The raw data are presented in Appendix E and are presented here, graphically, in Figures 9 and 10. These graphs are shown in larger scale in Appendix E.

These graphs and most of the analysis are based on the simplified regression polynomials, Eqs (20a) and (21a). The rounding error for the terms less than four decimal places is initially assumed small and simply added into the regression error. The danger of this assumption is discussed in the closing of the analysis.

Figure 9 represents the maximum countervalue available given a counterforce value requirement. Counterforce is displayed on the left, vertical axis, and countervalue on the horizontal axis. This graph also displays the optimal weapons combination to achieve this maximum CF for a given CV. The number of weapons is given along the right-vertical axis. For example, given a CV of 1190, the maximum CF is approximately 880. Likewise, the respective weapons set is 450, 536, 757, and 393. At the beginning of

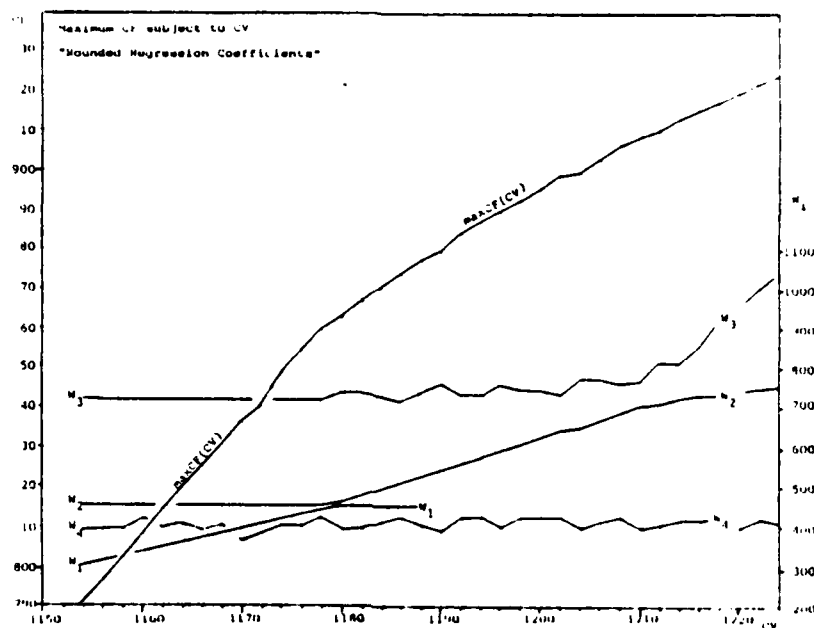


Fig 9. Maximum CF Subject to CV

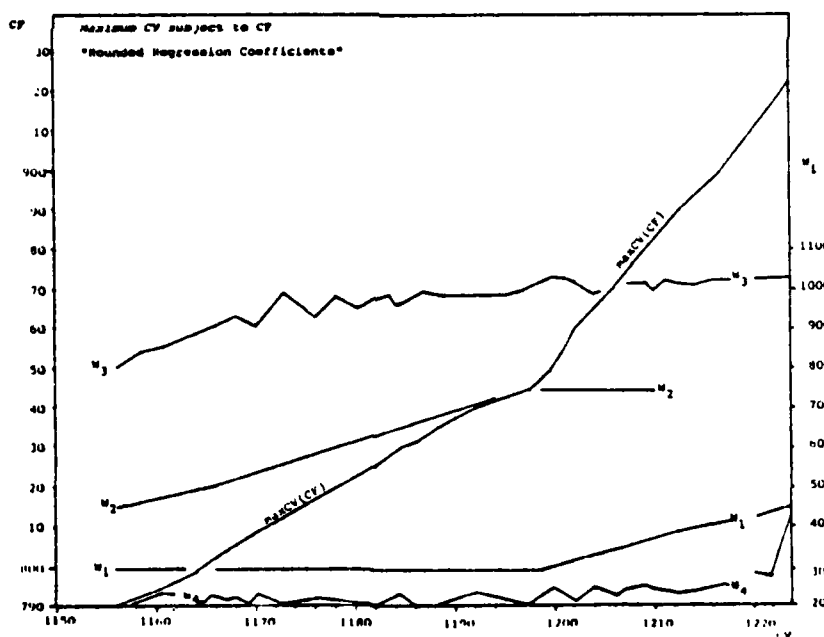


Fig 10. Maximum CV Subject to CF

the graph, weapons two and three are approximately constant at their lower limits of 450 and 720 respectively. Weapon one is steadily increasing to its maximum limit of 450. Weapon one reaches this upper limit at approximately 1180 CV. Interpretation of this figure is left for the analysis in the next section. Figure 10 is similar to Figure 9, but the roles of CV and CF are reversed. Thus, the graph shows the maximum CV available for a given CF. In this case, weapon one remains constant at the beginning of the graph while weapon two is steadily increasing towards its upper limit. At approximately 850 CF, weapon one begins increasing as weapon two has reached its limit. Both these figures are compared and discussed in the following section.

Analysis

This section is concerned with three areas. First, an explanation of the trade-off between MOE's and what causes the changes in this trade-off. Second, the determination of the causes for the force structure changes and, in particular, the apparent randomness of W_3 and W_4 values on the graphs. Finally, the danger involved in using the rounded-off, simplified regression equations rather than the full polynomials is discussed.

Figure 9 shows the relative trade-off between maximum CF and CV as approximately two regions. Both these regions are approximately linear. The first region runs from a CV value of approximately 1154 to 1178.

The trade-off between MOE's in this region is approximated by a straight-line function of slope, 2.9. Likewise, the second region runs approximately from 1180 to 1206 CV and is closely approximated by a straight-line of slope, 1.52. These approximate slopes may be accurately calculated by the ratio of the total derivatives of each of the regression polynomials, Eqs (20a) and (21a). These total derivatives are shown in Eqs (22) and (23) with their respective examples in Eqs (22a) and (23a).

$$dCF = (\partial CF / \partial W_1) * dW_1 + (\partial CF / \partial W_2) * dW_2 + (\partial CF / \partial W_3) * dW_3 + (\partial CF / \partial W_4) * dW_4 \quad (22)$$

$$dCF = (.4716) * dW_1 + ((.2955) - (.002) * W_2) * dW_2 + (.0289) * dW_3 + (.0279) * dW_4 \quad (22a)$$

$$dCV = \sum_i (\partial CV / \partial W_i) * dW_i \quad (23)$$

$$dCV = (.1658) * dW_1 + (.1198) * dW_2 + (.0308) * dW_3 + (.0019) * dW_4 \quad (23a)$$

Thus, the trade-off between MOE's is equal to the slope, $dMOE_1 / dMOE_2$.

From Chapter Three's data (Appendix E), the ratio of CF to CV changes, $\Delta CF / \Delta CV$, may be compared to the total derivative's ratio. For example, from a CV of 1162 to a CV of 1168, the $\Delta CF / \Delta CV = 2.8817$, which compares to the

ratio of total derivatives of CF to CV of 2.8766, ($dCF/dCV = 2.8766$). Likewise, the second region from CV 1180 to 1206 has a slope, $\Delta CF/\Delta CV$, equal to 1.53, which compares to the total derivative ratio of 1.51.

Since the change in weapons (dW_1) is the same for both the CF and CV total derivatives, the ratio of each coefficient may be compared separately if the other dW_1 's are relatively small. Thus, in this case, the dW_1 coefficient equals $.4716/.1658$ which equals 2.84. In the region 1162 CV to 1168 CV (Figure 9), only W_1 is actively changing. Thus, the trade-off between MOE's in this region is closely approximated by the dW_1 coefficients' ratio. The difference between the two ratios ($2.8766 - 2.84 = .0366$) is due to the small change in the other weapon systems (dW_i).

This technique, of course, becomes more difficult when the coefficients have cross terms or square terms such as the W_2 term. In this case, an average value of W_2 for the range of consideration must be applied in the calculations. Although this explains the relative trade-off or slope of the maximum CF given a CV line, it should not be inferred as the effectiveness of a weapon system to increase the maximizing MOE. Weapon four, for example, has a ratio of 14.68 for its MOE trade-off. However, the actual change in CF per weapon four is very small with its CF regression coefficient of .0279. Consequently, although the relative ratio may give a high rate of return in terms of one MOE to

another, it may have little importance as to the actual change in total value of the maximum MOE. This leads to the question of which weapons are the most effective in the optimal force structure to maximize a secondary MOE given the level of a primary constraining MOE.

Smith and Mellichamp's work (30) shows the independent regression polynomial coefficients used to determine the incremental increase in the MOE per weapon. Using this rationale, the maximizing MOE prefers the weapon system with the highest coefficient first. Thus, the increase in the MOE will be due to the maximizing MOE's regression coefficients and the amount of weapons available. The number of weapons available, however, becomes a function of the level of the constraining MOE. Thus, given a CV level, the operating domain is restricted as shown graphically with the contour lines. Operating within this restricted domain, the maximizing function attempts to trade-off the highest coefficient weapons system with the number available and compare its relative MOE increase to that of introducing some less efficient system that may change the number of weapons available. This is best shown in Eq (24).

$$CF(W_i) = \frac{\partial CF}{\partial W_i} * [\text{Number of } W_i \text{ available}] \quad (24)$$

$$CF(W_1) = (.4716) * \left[\frac{(CV - (.1198) * W_2 - (.0308) * W_3 - (.0019) * W_4)}{.1658} \right] \quad (25)$$

In Eq (24) the number of W_i available is simply the regression polynomial for the constraining MOE solved for that particular weapon system (W_i). Eq (25) shows the example of maximizing CF in terms of weapon system one. The maximizing algorithm (SUMT) must trade-off the introduction of a lesser efficient weapon system against the value of not decreasing the number of weapons available for contribution in Eq (25). This is numerically shown in Eqs (26), (27), and (28).

$$CF(W_1) \text{ gain} \geq CF(W_i) \text{ gain} \quad (26)$$

$$.4716 \left(\frac{.0308}{.1658} W_3 \right) = .188W_3 \geq .0289W_3 \quad (27)$$

$$.4716 \left(\frac{.0019}{.1658} W_4 \right) = .0116W_4 \not\geq .0279W_4 \quad (28)$$

When Eq (26) is true, then the algorithm will not increase W_i . Eq (27) is the gain to CF as a function of weapon one due to the level of weapon three as compared to the value of weapon three, as determined by its CF regression coefficient. In Eq (27) a .188 gain is determined in the CF equation due to weapon one, whereas only .0289 is due to weapon three. This case is not true, however, in Eq (28) where the gain for weapon one is only .0116 as compared to the actual increase of .0279 if weapon four is introduced. Therefore, the algorithm will choose to enter weapon four rather than leave it out in favor of the number of weapon

ones available. This preference for W_4 while W_1 is increasing is shown in Figure 9. In the W_1 increasing region, W_4 is at or near its upper limits while W_3 is at its lower limit.

It is due to this algorithm trade-off comparison between each domain system's relative MOE increase that the graph of the data appears erratic. This is especially true for the smaller coefficient values such as weapons three and four. In light of this erratic change, no attempt has been made to smooth the weapons' curves.

The section, thus far, has explained the trade-off and algorithm optimization of the force structure by using the simplified regression polynomials. The final point in this analysis section is to determine the error introduced by this simplification and its implications.

Figure 11 is a plot of max CF vs. CV using the full polynomials given in Eqs (20) and (21). A comparison of Figures 11 and 9 shows there is, indeed, a difference in not only trade-off value but also in the weapon systems' order and rate of entry.

The reason for the change in slope or trade-off is due to both a reduced value in CV and the inclusion of the square and cross terms, and also an increased value in CF due to this more accurate use of the square and cross terms. At first glance, this does not appear plausible for the case of CF. However, the rounding up of only the W_2^2

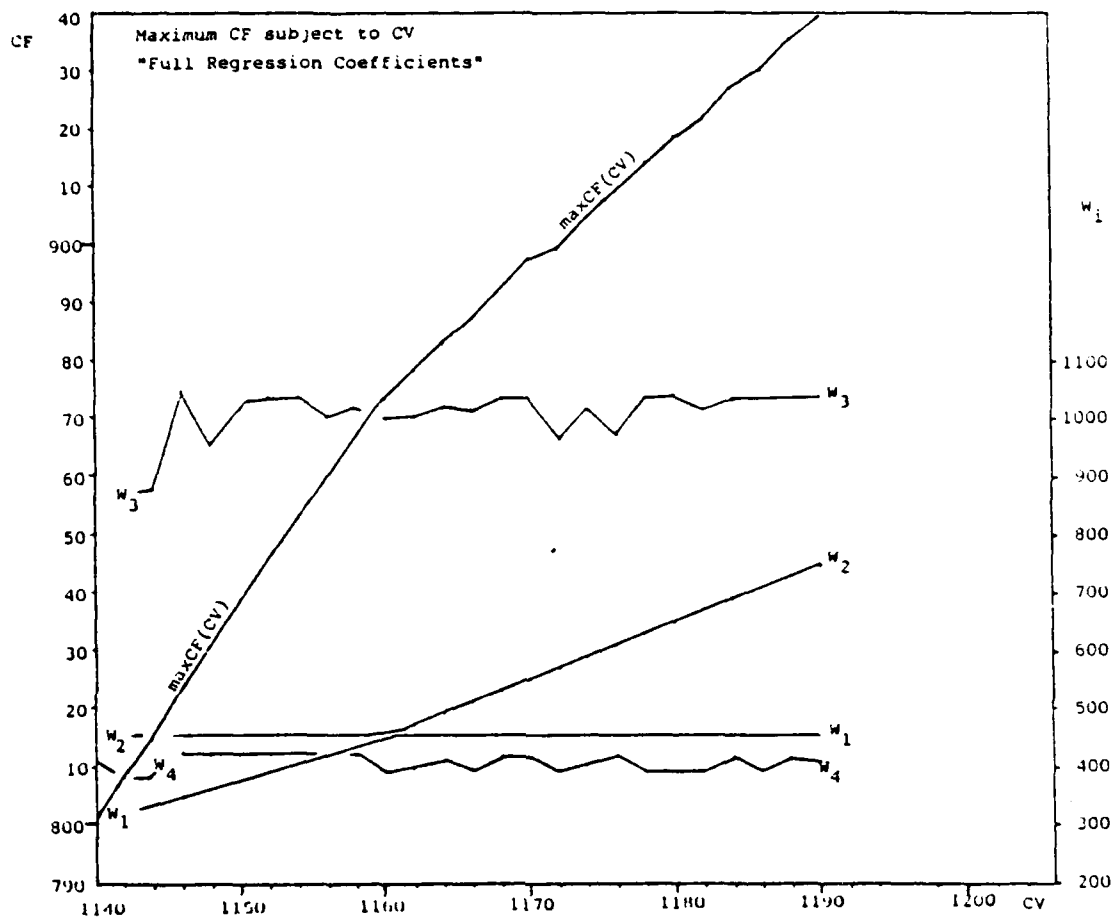


Fig 11. Maximum CF Subject to CV
Full Regression Coefficients

CF coefficient causes the change in maximum CF available from the original 939 down to a lesser value of 924 in the simplified equation. Thus, the rate of trade-off is less in a simplified case.

The change in the force structure due to the rate at which different systems are entering is not greatly affected by weapons one or two. However, there is a large change in weapon three. Again, this change is due to the trade-off done by the optimization technique when looking at weapon three's introduction in comparison to reducing the numbers of weapons one and two available.

Throughout the analysis, the only mention of the bias error listed in Table V has been in the minimum contour interval determination. Ignoring such a small error has little effect on the MOE trade-off calculation; however, it may affect the force structure numbers of the lesser efficient domain elements (weapons). Thus, the constraining numbers of weapons available should be recalculated to determine if two standard deviations of error significantly change the optimal n-tuples.

In conclusion, the analysis has found and been able to describe the relationship between measures of effectiveness and the optimal force structure. Likewise, it has shown that the original assumption of rounding to the fourth decimal place changes the overall structure of the problem.

The question of rounding the four digits originally arose in the research because the regression package used automatically rounded to four decimal places as seen in Appendix C. However, when the package did compute its residual error it had, in fact, maintained the values of those coefficients to the degree of accuracy that the computing device used. Thus, rather than using the SPSS automated covariance matrix, the problem could have been avoided by using the coded design.

Decision Tools

The above methodology should, with a high degree of accuracy, predict an optimal force to achieve a maximum measure of effectiveness given a constraint level of a second measure of effectiveness. Thus, the decision maker has, in Figure 11, the ability to see not only his optimal force structure, but how that force will change as changes occur in his MOE requirements.

So far this methodology has given the decision maker his best single domain set for a given constraint MOE. It is foolish to assume that in any analysis all the factors would be known or that some which were assumed to be constant in the operating environment may not have changed. In light of this, a choice of optimal solutions would probably be better suited as a tool for the decision maker. The SUNT program finds only a single optimal solution; however,

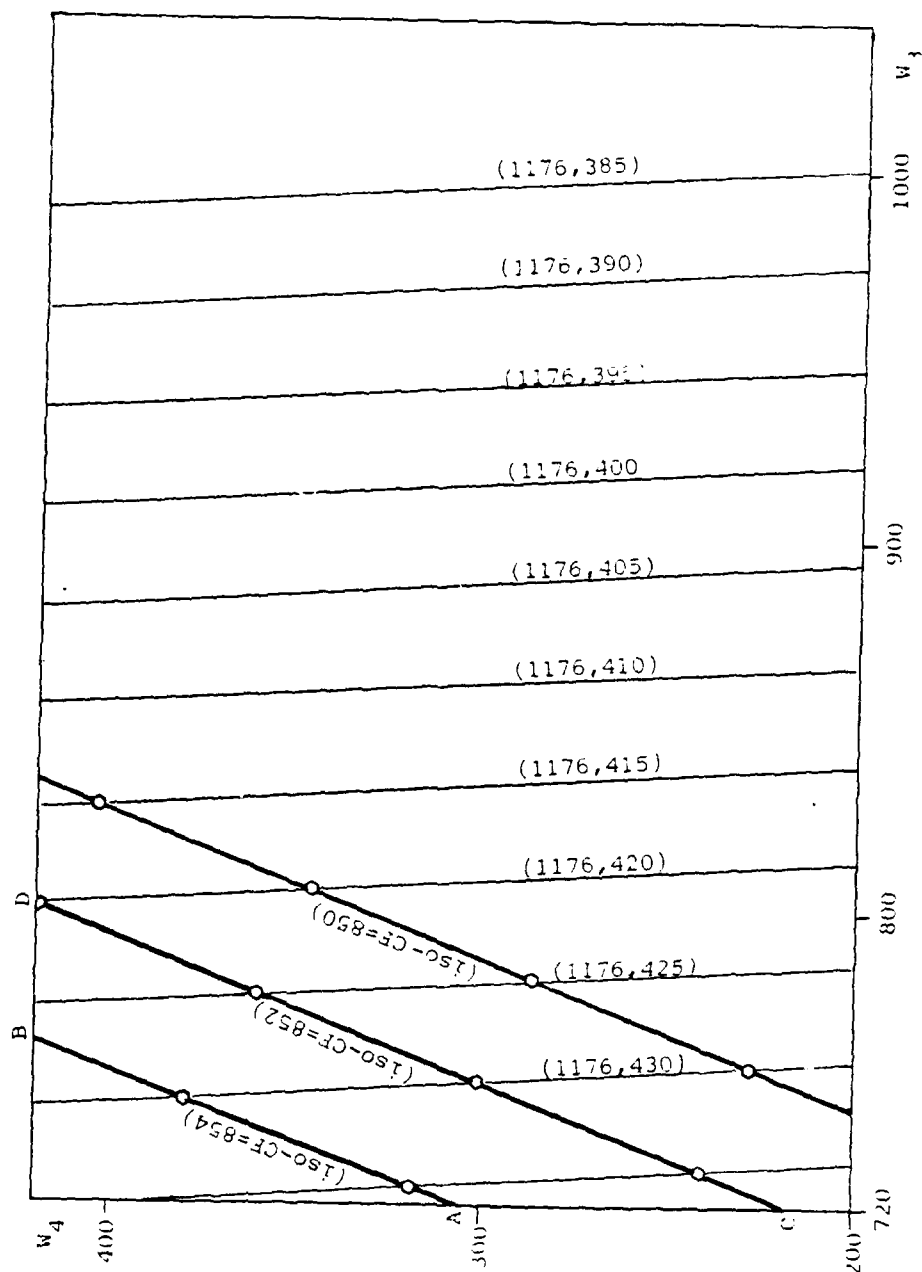


Fig 12. Multiple CF Solution Sets = 850, 852, 854 for $CV = 1176$

the graphic techniques applied in the opening of this chapter can also show a near optimal set of solutions. Figure 12 shows the overlay for CV 1176 with CF plots of 850, 852, and 854 CV. Lines connecting the common domain points for each CF value are shown in Figure 12 as iso-CF lines (heavy print lines), given the CV level of 1176. Thus, any of the force structures along one of the iso-CF lines will guarantee that CF value and the CV of 1176. For example, the line AB is a CF value of 854, given a CV value of 1176. This AB line contains domain ranges of W_1 from 436 to 427, W_3 from 720 to 765, and W_4 from 305 to 420. Although these alternatives have given up approximately 1.4 CF from the SUMT optimal solution of 355.4 CF, it may be of greater overall value to the decision maker to have these multiple options available to him. Thus, these lines offer the decision maker a trade-off from a single optimal solution to a set of multiple solutions.

Finally, the research has yet to consider the decision maker weighting the value of one MOE to another. However, if the decision maker did decide to weight the MOE's, it would have little change on the methodology as it has been applied. For example, if the decision maker decided that CF was twice as valuable as CV, the only change would be to multiply the total derivative ratio denominator by two. This is shown in Eqs (29a) and (29b) respectively.

Thus, the slope of the trade-off curve has decreased; however, the optimal force to create that curve is unchanged.

$$CF = 2CV \quad (29a)$$

$$\frac{dCF}{d(2 \cdot CV)} = \frac{dCF}{2 \cdot dCV} \quad (29b)$$

If, on the other hand, the decision maker were to change the value of the individual targets rather than the value of the overall MOE class, then this would require a complete re-sampling and running of the model.

Conclusion

This chapter contains the analysis portion of the research. The analysis began with a graphic technique to maximize the counterforce MOE given a countervalue constraint contour of 1176. This graphic presentation validated the optimization technique and the SUMT algorithm employed to calculate the constrained maxima. Following the single graphic case, the maximum CF trade-off given a CV level was analyzed for the entire experimental domain.

This analysis revealed the rate of trade-off to be the MOE's ratio of total derivatives ($dMOE_O/dMOE_C$). This maximized trade-off was determined by the optimal selection of domain elements (n-tuples). This selection was shown to be a complex trade-off between each element's ability to increase the maximized MOE ($\partial MOE_O/\partial d_i$) and the availability

of each element as determined by the constraint MOE

$$(d_i = f(MOE_c)).$$

Finally, the decision maker's use of the analysis was greatly enhanced by utilizing "near optimal" solution sets. These solutions were shown graphically to contain multiple n-tuples which achieve equal value MOE (iso-CF line, Figure 12). Thus, for a relatively small decrease from the SUMT optimal (less than one percent), the decision maker was given greater flexibility in his choice of operating domain.

The advantages of this analysis are further discussed in the concluding chapter.

CHAPTER FIVE

Concluding Remarks

Research Summary

This study presents the optimization methodology for a deterministic, mathematically modeled system as defined by the aspiration levels of one measure of effectiveness (goal/mission) and optimization of a second measure of effectiveness (MOE). The methodology's advantage over other multicriteria decision theories is its ability to consider large numbers of alternatives, show the MOE's comparisons throughout the range of alternatives, and use non-commensurate measures of effectiveness.

The research example demonstrates the methodology applied to a generic, aggregated military force structure problem defined by a linear programming arsenal exchange model. The example has two measures of effectiveness, counterforce (CF) and countervalue (CV). The system involves five weapon systems, of which four are varied over their plausible ranges, and ten target classes. The number of feasible force structure alternatives over these ranges exceed three billion. The optimal effectiveness of these alternatives is closely approximated (less than one percent error) by use of response surface methodology.

The employment of a central composite 3^k fractional factorial research design enabled the generation of a surface for each MOE from a minimal number of model runs. The

original design required 27 samples per surface (2). However, the deterministic model has no variance; thus, the samples are reduced to 25 per surface. With these two surfaces, CF and CV, the methodology then shows an appropriate interval to describe the optimal trade-off between the aspiration MOE and the second MOE. The interval selection criteria includes both the regression error and the change in the contour surface. With this selection of contours, the methodology then optimizes the second measure of effectiveness as constrained to the contour lines of the aspiration measure of effectiveness. These data are then plotted and displayed in Chapter Four of the analysis and also in Appendix E.

The analyses of these data give the decision maker a range of tools to gain further insight into the optimal operation of the system. The data map displays the decision maker's optimal system structure given his goal aspiration level. As new demands are made upon the system's measures of effectiveness, this map shows the required changes to each component of the system to achieve the higher MOE demands. The graphic techniques of Chapter Four offer the decision maker a choice of multiple solutions which very closely approximate the optimal trade-off value. For the example given, an aspiration/demand of 1176 CV, the decision maker has a wide range of force structures to choose from by simply reducing the CF value by 1.4 from the

optimal of 855.4 This reduction is less than two tenths of a percent from the optimal value. In consideration of the accuracy of the vulnerability determination and single-shot probabilities of kill used in the arsenal exchange model, this small reduction in the optimization process is minimal. However, this negligible reduction gives the decision maker greater flexibility in his choice of the system's operational structure.

Applications

Considering the number of deterministic optimization models in use today, this methodology should have widespread application when viewing multiple missions or roles of the system.

The military applications are several. The methodology application to force structure planning is well illustrated in the research example and Smith and Mellichamp's work (30). Beside force planning, the methodology may give insight as a measurement and design tool for weapon systems.

Many of today's weapon systems are required to perform multiple missions, some of which may be performed by several configurations of a weapon system or by different systems. This methodology may serve as a measurement tool to determine which configuration or system performs best in the dual mission. Likewise, the methodology may be applied as a design tool for the weapon systems. For instance, if a new bomber was being designed, it may have a strategic

penetration mission (aspiration MOE) and also a search and destroy mission (second MOE). The operating domain would be the flight characteristics and capability of the bomber. This methodology would choose the best selection of operational capabilities to achieve the aspired penetration goal and also maximize the second mission of search and destroy. It would also show how these capabilities would have to be changed to meet new mission requirements. Thus, this methodology offers a planning tool for new systems as well as total forces.

A larger scale application may be in force structure as seen in an arms reduction or limitations agreement. In this case, if both parties were able to represent their respective capabilities in a response surface, then each would be able to offer and counteroffer an optimal selection of weapon systems from their particular arsenal that equated to equal levels of damage expectancy for the agreement.

Further Research

This methodology was originally proposed for the comparison of non-commensurate goals. As the study evolved, it was obvious the scope would not allow the comparison of more than two surfaces at a time. As such, further research is required for the comparison of three or more measures of effectiveness.

The dual measure of merit case has the advantage of not changing the optimal solution domain regardless of the

decision maker's weighting of the MOE's. However, in the case of three or more measures of merit, this weighting may determine the interval contour pattern as it restricts the operating domain of the maximized MOE under consideration.

Another approach may be to use the dual measure of effectiveness and then project each maximized function into a next higher dimension and find a third response surface common to the two optimization processes. For example, find the maximum A subject to C and the maximum B subject to C. Then, determine the maximum C as a function of $\max A(C)$ and $\max B(C)$.

APPENDIX A

Aggregated Arsenal Exchange Model

This appendix details the aggregated arsenal exchange model employed in this study. The model was developed with the expert assistance of Captain Robert Bunnell. Its purpose is to generate three different response surfaces for the methodology example. As such, its complexity is minimal in comparison to other arsenal exchange models.

Chapter Three's Tables I and II are repeated below as a description of the target class and weapon class characteristics.

TABLE I

Weapon System Characteristics

	W ₁	W ₂	W ₃	W ₄	W ₅
CEP	.200	.150	.240	.600	.200
Warheads/Weapon	1	3	10	4	6
Yield	2.0	.17	.05	.35	.20
Reliability	.85	.85	.85	.85	.85
Daily Alert	.85	.85	.85	.33	.33
Max Warheads/Type	450	750	1040	424	360

TABLE II
Target Base Characteristics

Type	Number	Vulnerability Number	Diameter
CV1	140	24Q0	.56
CV2	215	13P1	.49
CV3	430	31P6	.00
CV4	520	23Q0	.31
CF1	450	35P7	.00
CF2	1000	52P8	.00
CF3	200	39P0	.00
CF4	20	22P1	.36
CF5	150	11P0	.00
CF6	100	10Q1	.79

As mentioned in Chapter Three, the vulnerability number characterizes a target's vulnerability to its most susceptible threat (i.e., overpressure, dynamic pressure, etc.). The first two digits are its level of susceptibility to the threat. The alpha character is the most threatening type of effect for the target in question. The last number is the target's sensitivity to the duration of the particular weapon's effect. Thus, with five alpha numeric characters,

the target's worst case vulnerability is defined.

Likewise, the weapon's ability to create the effects used in the target VNTK number is calculated from its yield in megatons, circular error probable (CEP), and the probability of reaching the target (reliability). These effects are determined in a cumulative, lognormal damage function (1:3).

The target and weapon characteristics are combined in accordance with the physical effects equations of the Defense Intelligence Agency's Physical Vulnerability Branch (1:34) to determine a single-shot probability of survival (SSP_s) for each target when attacked by each weapon. Thus, for this example of five weapon types, five SSP_s are calculated for each target. The transformation from the VNTK number and weapon characteristic to SSP_s is accomplished by a computer program used in the Bunnell and Takacs study (5).

The SSP_s are converted into single-shot probabilities of kill (SSP_k) for each target and weapon combination, Eq (30).

$$SSP_k = 1 - SSP_s \quad (30)$$

Likewise, the single-shot probability of kill is calculated for two weapons of the same type against each target type, Eq (31).

$$SSP_k'' = 1 - (SSP_s)^2 \quad (31)$$

Thus, each target now has five pairs of SSP_k 's associated with it - one for single strike and one for two strikes by the same weapon.

The arsenal exchange is modeled using linear programming. The model uses the SSP_k 's and SSP_k'' 's as the coefficient for the objective function. The number of targets struck is the variable for the tableau. The LP algorithm maximizes the sum of damage possible as determined by the SSP_k coefficients times the number of targets struck. Eq (32) depicts this example objective function.

$$\sum_{w=1}^5 \sum_{k=1}^{10} SSP_{k_w} \cdot T_k + SSP_{k_w}'' \cdot T_k'' \quad (32)$$

where

k = target struck class
 w = weapon class

In this tableau, a separate variable is used for a target struck twice by a particular weapon class.

The first ten rows of the tableau define the sum of each class of targets available, Eq (33).

$$\begin{aligned} \text{row}(i) &= \# \text{ of targets in class } (i) \\ &= \sum_{j=1}^{50} T_{i(2j-1)} + T_{i(2j)}'' \end{aligned} \quad (33)$$

where

$(2j-1)$ = odd columns up to 99
 $(2j)$ = even columns up to 100
 i = 1 through 10

AD-A141 169

AN OPTIMIZATION METHODOLOGY FOR MULT--CRITERIA

2/2

COMPARISON USING RESPONSE. (U) AIR FORCE INST OF TECH

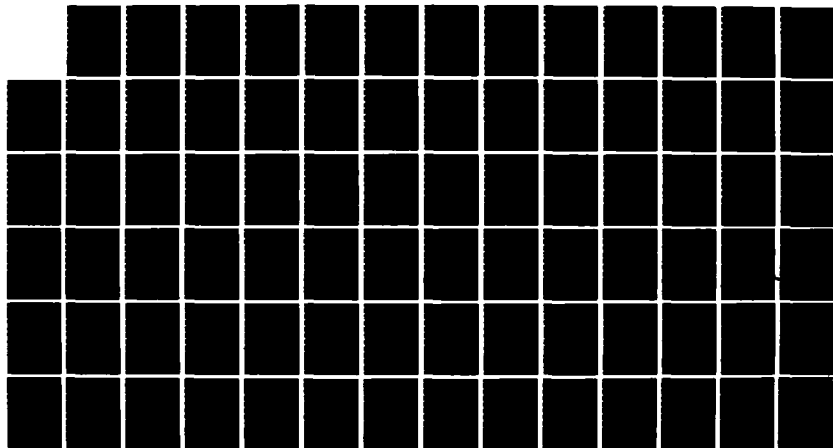
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI. R E GRANEY

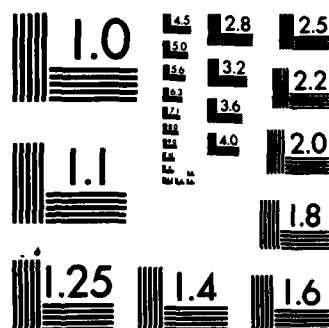
UNCLASSIFIED

MAR 84 AFIT/GST/OS/84M-11

F/G 12/1

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Rows 11 through 15 define the sum of each class of weapons available, Eq (34).

$$\begin{aligned} \text{row } (i) &= \# \text{ weapons in class } (i) \\ &= \sum_{j=1}^{50} T_{i(2j-1)} + 2 \cdot T_{i(2j)} \end{aligned}$$

(34)

where

$$\begin{aligned} (2j-1) &= \text{odd columns up to 99} \\ (2j) &= \text{even columns up to 100} \\ i &= 11 \text{ through } 15 \end{aligned}$$

The difference between the CF, CV, and AF models is in the objective function. The objective variables, T_{0j} , which are designated CF or CV are given a coefficient SSP_k of zero if not in the exchange under consideration. Thus, to run a CF exchange, all CV targets are given an SSP_k of zero.

Since the model was to have multiple runs for the weapons available in rows 11 through 15, a program was written which compiled a tableau for each MOE (CF, CV, AF). This program, CREATE2, is an interactive program which requests the user to enter the right-hand-side values (RHS) for the number of weapons available in each class (rows 11 through 15). The program then requests if any other tableau values should be changed from the initial values already constructed into the tableau. Upon completion of these two data entry steps, the program sends

the tableau for each option to local files AF, CF, and CV, respectively. The tableau format is determined by the linear programming package "Multi-Purpose Optimization System (MPOS)." The source code for this Fortran Five program follows.

```

PROGRAM CREATED
DIMENSION AIJ(0:15,0:100),CFIJ(0:15,0:100),C(0:100:15,0:100)

C
C
C
C   THIS PROGRAM CREATES THE DATA MATRIX FOR AN A-10/STAFF A-10/STAFF
C   EXCHANGE MODEL TO BE SOLVED BY MPOS. THE MODEL HAS TEN
C   TARGET CATEGORIES AND FIVE WEAPON TYPES. THE MODEL ASSUMES
C   A SINGLE TYPE WEAPON AGAINST A PARTICULAR TARGET. THE OBJECTIVE
C   OBJECTIVE(ROW 0) MAXIMIZES THE DAMAGE INFLECTED ON THE
C   TOTAL TARGET BASE. THE MODEL ALLOWS A SINGLE WEAPON OR A
C   PAIR OF SAID WEAPONS TO BE ALLOCATED TO A SINGLE TARGET.
C
C   NOTE: THE RIGHT-HAND SIDE VALUES(RHS) ARE IN COLUMN 16 AND
C   THE OBJECTIVE COEFF. ARE IN ROW ZERO
C
C
C
C
C   INITIALIZE MATRIX AIJ TO ALL ZEROS.
C
      DO 200, J=0,100
        DO 100, I=0,15
          AIJ(I,J)= 0.0
100    CONTINUE
200    CONTINUE
C
C
C   BUILDING TARGET CONSTRAINT SET(ROWS 1 TO 10)
C
      DO 400, II=0,80,20
        DO 300, I=1,10
          AIJ(I,(2*I+II))= 1.0
          AIJ(I,(2*I-1+II))= 1.0
300    CONTINUE
400    CONTINUE
C

```

```

C
C      BUILD WEAPON CONSTRAINT SET(ROWS 11 TO 15)
C
C      JA=1
C      JB=19
C
C      DO 800, I=11,15
C          DO 600, J=JA,JB,2
C              AIJ(I,J)= 1.0
C              AIJ(I,(J+1))=2.0
600      CONTINUE
C
C          JA=JA+20
C          JB=JB+20
C
C      800  CONTINUE
C
C      SET RHS (COL.0 ROWS 0 THRU 10) FOR OBJECTIVE AND NUMBER OF
C          TARGETS IN EACH CATEGORY.
C
C          AIJ(0,0)= 0.0
C          AIJ(1,0)= 140.0
C          AIJ(2,0)= 215.0
C          AIJ(3,0)= 450.0
C          AIJ(4,0)= 1000.0
C          AIJ(5,0)= 200.0
C          AIJ(6,0)= 20.0
C          AIJ(7,0)= 150.0
C          AIJ(8,0)= 100.0
C          AIJ(9,0)= 430.0
C          AIJ(10,0)= 520.0
C
C
C      SET PK'S IN OBJECTIVE FUNCTION(ROW 0, COLS. 1 TO 100)
C
C          AIJ(0,1)= 0.84797
C          AIJ(0,2)= 0.97689
C          AIJ(0,3)= 0.85000
C          AIJ(0,4)= 0.97750
C          AIJ(0,5)= 0.83249
C          AIJ(0,6)= 0.97194
C          AIJ(0,7)= 0.47032
C          AIJ(0,8)= 0.71944
C          AIJ(0,9)= 0.67217
C          AIJ(0,10)= 0.89253
C          AIJ(0,11)= 0.84999
C          AIJ(0,12)= 0.97750
C          AIJ(0,13)= 0.85000
C          AIJ(0,14)= 0.77750
C          AIJ(0,15)= 0.85000

```


AIJ(0,16)=	0.97750
AIJ(0,17)=	0.84660
AIJ(0,18)=	0.97647
AIJ(0,19)=	0.84979
AIJ(0,20)=	0.97744
AIJ(0,21)=	0.76716
AIJ(0,22)=	0.94579
AIJ(0,23)=	0.85000
AIJ(0,24)=	0.97750
AIJ(0,25)=	0.61073
AIJ(0,26)=	0.84847
AIJ(0,27)=	0.15429
AIJ(0,28)=	0.28478
AIJ(0,29)=	0.37563
AIJ(0,30)=	0.61016
AIJ(0,31)=	0.83104
AIJ(0,32)=	0.97145
AIJ(0,33)=	0.85000
AIJ(0,34)=	0.97750
AIJ(0,35)=	0.85000
AIJ(0,36)=	0.97750
AIJ(0,37)=	0.71782
AIJ(0,38)=	0.92038
AIJ(0,39)=	0.81872
AIJ(0,40)=	0.96714
AIJ(0,41)=	0.44306
AIJ(0,42)=	0.68982
AIJ(0,43)=	0.84164
AIJ(0,44)=	0.97492
AIJ(0,45)=	0.14544
AIJ(0,46)=	0.26973
AIJ(0,47)=	0.02663
AIJ(0,48)=	0.05255
AIJ(0,49)=	0.08063
AIJ(0,50)=	0.15475
AIJ(0,51)=	0.51804
AIJ(0,52)=	0.76771
AIJ(0,53)=	0.84982
AIJ(0,54)=	0.97744
AIJ(0,55)=	0.84097
AIJ(0,56)=	0.97471
AIJ(0,57)=	0.22121
AIJ(0,58)=	0.39349
AIJ(0,59)=	0.52353
AIJ(0,60)=	0.77298
AIJ(0,61)=	0.36172
AIJ(0,62)=	0.59260
AIJ(0,63)=	0.82551
AIJ(0,64)=	0.96955
AIJ(0,65)=	0.11517
AIJ(0,66)=	0.21708

```

AIJ(0,67)= 0.02402
AIJ(0,68)= 0.04747
AIJ(0,69)= 0.04995
AIJ(0,70)= 0.09741
AIJ(0,71)= 0.40469
AIJ(0,72)= 0.64561
AIJ(0,73)= 0.84494
AIJ(0,74)= 0.97596
AIJ(0,75)= 0.83678
AIJ(0,76)= 0.97336
AIJ(0,77)= 0.17125
AIJ(0,78)= 0.31317
AIJ(0,79)= 0.40083
AIJ(0,80)= 0.64099
AIJ(0,81)= 0.72773
AIJ(0,82)= 0.92587
AIJ(0,83)= 0.85000
AIJ(0,84)= 0.97750
AIJ(0,85)= 0.48420
AIJ(0,86)= 0.73395
AIJ(0,87)= 0.10270
AIJ(0,88)= 0.19496
AIJ(0,89)= 0.26052
AIJ(0,90)= 0.45317
AIJ(0,91)= 0.80019
AIJ(0,92)= 0.96008
AIJ(0,93)= 0.85000
AIJ(0,94)= 0.97750
AIJ(0,95)= 0.85000
AIJ(0,96)= 0.97750
AIJ(0,97)= 0.60865
AIJ(0,98)= 0.84685
AIJ(0,99)= 0.78184
AIJ(0,100)= 0.95241

```

C
C
C
C
C
C

RHS VALUES REQUESTED FOR EACH WEAPON CATEGORY.
ANY COEFFICIENTS MAY BE CHANGED IN THIS SECTION. THE SESSION
IS TERMINATED WHEN THREE ZEROS ARE ENTERED.

```

900 PRINT*, 'INPUT ROW NUMBER, COLUMN NUMBER, COEFFICIENT: (I, J, COEFF)'
PRINT*, 'FOR EACH WEAPON RHS AND ANY OTHER COEFFICIENT TO BE'
PRINT*, 'CHANGED. ENTER ONLY ONE SET AT A TIME. TO TERMINATE'
PRINT*, 'ENTER THREE ZEROS (0 0 0)'
READ*, I, J, C
IF (I.EQ.0.AND.J.EQ.0.AND.C.EQ.0) GO TO 902
AIJ(I, J)=C
GOTO 900

```

```

C
902 PRINT*, 'ENTER THE RHS WEAPON ALLOCATION VALUES'
    DO 46, I=11,15
        READ*, C
        AIJ(I,0)=C
    46 CONTINUE
C
950 PRINT*, 'THE FILE CONTAINING YOUR ARSENAL MODEL MATRIX'
    PRINT*, 'IS CALLED-MN FOR ALL FORCES AND CV/CF FOR OPTIONS.'
C
C
C
C ARSENAL MATRIX WRITEN TO FILE MTX. ARSENAL MATRIX ALSO
C COPIED FOR CV AND CF OPTIONS.
C
    OPEN(12, FILE='MN')
    REWIND 12
    DO 954, I=0,15
        DO 952, J=0,100
            IF(AIJ(I,J).NE.0.0) WRITE(12,951) I,J,AIJ(I,J)
951 FORMAT(2X,I3,2X,I3,2X,F10.5)
            CFIJ(I,J)=AIJ(I,J)
            CVIJ(I,J)=AIJ(I,J)
C
952 CONTINUE
954 CONTINUE
    CLOSE(12)
C
C COUNTER-FORCE MATRIX: SET PK=0.0 FOR CV TARGETS IN OBJECTIVE
C
    OPEN(11, FILE='CF')
    REWIND 11
    DO 13, II=0,80,20
        DO 11, I=1,4
            CFIJ(0,I+II)= 0.0
11 CONTINUE
        DO 12, I=17,20
            CFIJ(0,I+II)= 0.0
12 CONTINUE
13 CONTINUE
C
C
C
C COUNTER-VALUE MATRIX: SET PK=0.0 FOR CF TARGETS IN OBJECTIVE
C
    OPEN(13, FILE='CV')
    REWIND 13
    DO 16, II=0,80,20
        DO 15, I=5,16
            CVIJ(0,I+II)= 0.0
15 CONTINUE
16 CONTINUE
C

```

```

C
      DO 21, I=0,15
      DO 20,J=0,100
      IF(CFIJ(I,J).NE.0.0) WRITE(11,22) I,J,CFIJ(I,J)
      IF(CVIJ(I,J).NE.0.0) WRITE(13,29) I,J,CVIJ(I,J)
20      CONTINUE
21      CONTINUE
29      FORMAT(2X,I3,2X,I3,2X,F10.5)
C
      CLOSE(11)
      CLOSE(13)
C
      STOP
      END
*****

```

The linear programming package chosen to optimize the tableaux is the Multi-Purpose Optimization system. Thanks to the work of fellow student, James Cooke, several flaws in the MPOS options were discovered prior to this study. Thus, the MIMIT option was chosen (7:38). This option solves a "sequence of primal and/or dual iterations," (7:38.1). Samples of the MPOS control cards for each MOE and a sample MPOS output are given below.

CF Control cards:

```

*THIS IS THE CONTROL CARD DECK FOR MPOS AEN
TITLE
ARSENAL EXCHANGE PROBLEM
MIMIT
VARIABLES
X1 TO X100
PACKED
MAXIMIZE
CONSTRAINTS 15
+++++
FORMAT
(2X,I3,2X,I3,2X,F10.5)
REWIND
READ CF
OPTIMIZE

```

CV Control cards:

```
*THIS IS THE CONTROL CARD DECK FOR MPOS AEM
TITLE
ARSENAL EXCHANGE PROBLEM
MINIT
VARIABLES
X1 TO X100
PACKED
MAXIMIZE
CONSTRAINTS 15
+++++
FORMAT
(2X,I3,2X,I3,2X,F10.5)
REWIND
READ CV
OPTIMIZE
```

AF Control cards:

```
*THIS IS THE CONTROL CARD DECK FOR MPOS AEM
TITLE MCN
ARSENAL EXCHANGE PROBLEM
MINIT
VARIABLES
X1 TO X100
PACKED
MAXIMIZE
CONSTRAINTS 15
+++++
FORMAT
(2X,I3,2X,I3,2X,F10.5)
REWIND
READ MN
OPTIMIZE
```

list,f=ocf

1 MPOS VERSION 4.0 NORTHWESTERN UNIVERSITY

```
*****
*                                     *
*               M P O S               *
*                                     *
*               VERSION 4.0           *
*                                     *
* MULTI-PURPOSE OPTIMIZATION SYSTEM *
*                                     *
*****
```

***** PROBLEM NUMBER 1 *****

```
*THIS IS THE CONTROL CARD DECK FOR MPOS AEM
TITLE
ARSENAL EXCHANGE PROBLEM
MINIT
VARIABLES
X1 TO X100
PACKED
MAXIMIZE
CONSTRAINTS 15
+++++
FORMAT
(2X,I3,2X,I3,2X,F10.5)
REWIND
READ OF
OPTIMIZE
```

1 MPOS VERSION 4.0 NORTHWESTERN UNIVERSITY

```
*****
* PROBLEM NUMBER 1 *
*****
```

```
USING MINIT
ARSENAL EXCHANGE PROBLEM
```

PROBLEM INPUT SUMMARY

CONSTRAINTS	VARIABLES	NON-ZEROS	PARAMETERS	BOUNDS
EUS= 0	INT= 0	NUMBER= 200	TOL= .100E-07	
LES= 15	TOTAL= 100	PERCENT= 13.33	EPS=DEFAULT	
GES= 0	NOUB= 0		LIMIT=DEFAULT	
TOTAL= 15	NOLB= 0		RSCALE= .100E+01	

VARIABLE TABLE

1 - X1	2 - X2	3 - X3	4 - X4	5 - X5	6 - X6
7 - X7	8 - X8	9 - X9	10 - X10	11 - X11	12 - X12
13 - X13	14 - X14	15 - X15	16 - X16	17 - X17	18 - X18
19 - X19	20 - X20	21 - X21	22 - X22	23 - X23	24 - X24
25 - X25	26 - X26	27 - X27	28 - X28	29 - X29	30 - X30
31 - X31	32 - X32	33 - X33	34 - X34	35 - X35	36 - X36
37 - X37	38 - X38	39 - X39	40 - X40	41 - X41	42 - X42
43 - X43	44 - X44	45 - X45	46 - X46	47 - X47	48 - X48
49 - X49	50 - X50	51 - X51	52 - X52	53 - X53	54 - X54
55 - X55	56 - X56	57 - X57	58 - X58	59 - X59	60 - X60
61 - X61	62 - X62	63 - X63	64 - X64	65 - X65	66 - X66
67 - X67	68 - X68	69 - X69	70 - X70	71 - X71	72 - X72
73 - X73	74 - X74	75 - X75	76 - X76	77 - X77	78 - X78
79 - X79	80 - X80	81 - X81	82 - X82	83 - X83	84 - X84
85 - X85	86 - X86	87 - X87	88 - X88	89 - X89	90 - X90
91 - X91	92 - X92	93 - X93	94 - X94	95 - X95	96 - X96
97 - X97	98 - X98	99 - X99	100 - X100		

INPUT TRANSLATION TIME = .1810 SECONDS

DATA STORAGE REQUIREMENTS ARE 004264 OCTAL
 FIELD LENGTH FOR THIS PROBLEM 050000 OCTAL
 1 MPOS VERSION 4.0 NORTHWESTERN UNIVERSITY

 * PROBLEM NUMBER 1 *

USING MINIT
 ARSENAL EXCHANGE PROBLEM

ENTERING PHASE II

ITERATION	1	IN VAR-	5	OUT VAR-	103	-ZMIN=	374.621
ITERATION	2	IN VAR-	34	OUT VAR-	107	-ZMIN=	521.246
ITERATION	3	IN VAR-	30	OUT VAR-	105	-ZMIN=	643.278
ITERATION	4	IN VAR-	96	OUT VAR-	108	-ZMIN=	741.028
ITERATION	5	IN VAR-	47	OUT VAR-	113	-ZMIN=	764.462
ITERATION	6	IN VAR-	32	OUT VAR-	106	-ZMIN=	783.891
ITERATION	7	IN VAR-	88	OUT VAR-	115	-ZMIN=	799.488
ITERATION	8	IN VAR-	56	OUT VAR-	96	-ZMIN=	813.379
ITERATION	9	IN VAR-	68	OUT VAR-	104	-ZMIN=	820.024
ITERATION	10	IN VAR-	28	OUT VAR-	112	-ZMIN=	821.211
ITERATION	11	IN VAR-	74	OUT VAR-	68	-ZMIN=	853.040
ITERATION	12	IN VAR-	52	OUT VAR-	34	-ZMIN=	857.705
ITERATION	13	IN VAR-	92	OUT VAR-	32	-ZMIN=	858.098
ITERATION	14	IN VAR-	48	OUT VAR-	52	-ZMIN=	858.847
ITERATION	15	IN VAR-	27	OUT VAR-	28	-ZMIN=	862.888
ITERATION	16	IN VAR-	87	OUT VAR-	47	-ZMIN=	864.347
ITERATION	17	IN VAR-	76	OUT VAR-	114	-ZMIN=	864.592
ITERATION	18	IN VAR-	54	OUT VAR-	56	-ZMIN=	864.604
ITERATION	19	IN VAR-	6	OUT VAR-	111	-ZMIN=	864.604
ITERATION	20	IN VAR-	7	OUT VAR-	6	-ZMIN=	864.604
ITERATION	21	IN VAR-	25	OUT VAR-	27	-ZMIN=	897.598
ITERATION	22	IN VAR-	90	OUT VAR-	5	-ZMIN=	904.335
ITERATION	23	IN VAR-	86	OUT VAR-	88	-ZMIN=	906.086
ITERATION	24	IN VAR-	47	OUT VAR-	90	-ZMIN=	914.463
ITERATION	25	IN VAR-	26	OUT VAR-	87	-ZMIN=	923.882
ITERATION	26	IN VAR-	9	OUT VAR-	25	-ZMIN=	931.688
ITERATION	27	IN VAR-	52	OUT VAR-	92	-ZMIN=	932.666

1 MPOS VERSION 4.0 NORTHWESTERN UNIVERSITY

 * PROBLEM NUMBER 1 *

USING MINIT
 ARSENAL EXCHANGE PROBLEM

SUMMARY OF RESULTS

VARIABLE NO.	VARIABLE NAME	BASIC NON-BASIC	ACTIVITY LEVEL	OPPORTUNITY COST	ROW NO.
1	X1	NB	--	.4696100	
2	X2	NB	--	.9392200	
3	X3	NB	--	.4696100	
4	X4	NB	--	.9392200	
5	X5	NB	--	.0779900	
6	X6	NB	--	.4081500	
7	X7	B	355.0000000	--	
8	X8	NB	--	.2204900	
9	X9	B	95.0000000	--	
10	X10	NB	--	.2492500	
11	X11	NB	--	.3354900	
12	X12	NB	--	.5775900	
13	X13	NB	--	.5452100	
14	X14	NB	--	.8873200	
15	X15	NB	--	.5426100	
16	X16	NB	--	.8847200	
17	X17	NB	--	.4696100	
18	X18	NB	--	.9392200	
19	X19	NB	--	.4696100	
20	X20	NB	--	.9392200	
21	X21	NB	--	.2038000	
22	X22	NB	--	.4076000	
23	X23	NB	--	.2038000	
24	X24	NB	--	.4076000	
25	X25	NB	--	.0339400	
26	X26	B	270.0000000	--	
27	X27	NB	--	.0502200	
28	X28	NB	--	.1235300	
29	X29	NB	--	.0307300	
30	X30	B	105.0000000	--	
31	X31	NB	--	.0886300	
32	X32	NB	--	.1520200	
33	X33	NB	--	.2794000	
34	X34	NB	--	.3557000	
35	X35	NB	--	.2768000	
36	X36	NB	--	.3531000	
37	X37	NB	--	.2038000	
38	X38	NB	--	.4076000	
39	X39	NB	--	.2038000	
40	X40	NB	--	.4076000	
41	X41	NB	--	.0259200	
42	X42	NB	--	.0518400	
43	X43	NB	--	.0259200	
44	X44	NB	--	.0518400	
45	X45	NB	--	.3213500	
46	X46	NB	--	.2229800	

SUMMARY OF RESULTS

VARIABLE NO.	VARIABLE NAME	BASIC NON-BASIC	ACTIVITY LEVEL	OPPORTUNITY COST	PON NO
47	X47	B	630.0000000	--	
48	X48	B	7.0000000	--	
49	X49	NB	--	.1478500	
50	X50	NB	--	.0996500	
51	X51	NB	--	.2237500	
52	X52	B	20.0000000	--	
53	X53	NB	--	.1017000	
54	X54	B	94.0000000	--	
55	X55	NB	--	.1079500	
56	X56	NB	--	.0001300	
57	X57	NB	--	.0259200	
58	X58	NB	--	.0518400	
59	X59	NB	--	.0259200	
60	X60	NB	--	.0518400	
61	X61	NB	--	.0251800	
62	X62	NB	--	.0503600	
63	X63	NB	--	.0251800	
64	X64	NB	--	.0503600	
65	X65	NB	--	.3508800	
66	X66	NB	--	.2741500	
67	X67	NB	--	.0018700	
68	X68	NB	--	.0036000	
69	X69	NB	--	.1777900	
70	X70	NB	--	.1555100	
71	X71	NB	--	.3363600	
72	X72	NB	--	.1206200	
73	X73	NB	--	.1058400	
74	X74	B	56.0000000	--	
75	X75	NB	--	.1114000	
76	X76	B	100.0000000	--	
77	X77	NB	--	.0251800	
78	X78	NB	--	.0503600	
79	X79	NB	--	.0251800	
80	X80	NB	--	.0503600	
81	X81	NB	--	.1465400	
82	X82	NB	--	.2930800	
83	X83	NB	--	.1465400	
84	X84	NB	--	.2930800	
85	X85	NB	--	.1032100	
86	X86	B	180.0000000	--	
87	X87	NB	--	.0445500	
88	X88	NB	--	.0988300	
89	X89	NB	--	.0885800	
90	X90	NB	--	.0424700	
91	X91	NB	--	.0622200	
92	X92	NB	--	.0488700	

 * PROBLEM NUMBER 1 *

USING MINIT
 ARSENAL EXCHANGE PROBLEM

SUMMARY OF RESULTS

VARIABLE NO.	VARIABLE NAME	BASIC NON-BASIC	ACTIVITY LEVEL	OPPORTUNITY COST	ROW NO.
93	X93	NB	--	.2221400	
94	X94	NB	--	.2411800	
95	X95	NB	--	.2195400	
96	X96	NB	--	.2385800	
97	X97	NB	--	.1465400	
98	X98	NB	--	.2930800	
99	X99	NB	--	.1465400	
100	X100	NB	--	.2930800	
101	--SLACK	B	140.0000000	--	(1)
102	--SLACK	B	215.0000000	--	(2)
103	--SLACK	NB	--	.4408700	(3)
104	--SLACK	NB	--	.0007100	(4)
105	--SLACK	NB	--	.2025600	(5)
106	--SLACK	NB	--	.7158700	(6)
107	--SLACK	NB	--	.9256000	(7)
108	--SLACK	NB	--	.9230000	(8)
109	--SLACK	B	430.0000000	--	(9)
110	--SLACK	B	520.0000000	--	(10)
111	--SLACK	NB	--	.4696100	(11)
112	--SLACK	NB	--	.2038000	(12)
113	--SLACK	NB	--	.0259200	(13)
114	--SLACK	NB	--	.0251800	(14)
115	--SLACK	NB	--	.1465400	(15)

MAXIMUM VALUE OF THE OBJECTIVE FUNCTION = 932.665560

CALCULATION TIME WAS .0720 SECONDS FOR 27 ITERATIONS.
 2TOTAL TIME FOR THIS PROBLEM WAS .359 SECONDS

APPENDIX B

Research Design

The research design chosen for this study is a three-level, 3^k , fractional factorial design (2). It is a four-factor design which requires twenty-seven samples for the model with variance. For this study, the LP model has no variance; therefore, the repetitions of the center point sample are not required. Thus, only twenty-five samples are required.

The design is shown with its respective sample values in the following examples. The first example is the uncoded design and each of the respective sampled MOE's (CF, CV, AF). This design was used for the revised lower limits of 300, 450, 720, and 200 for each W_i respectively. The remaining examples are in coded format (+1, 0, -1). This format assigns one for the upper limit of the variable, zero for the mid-range of the limit, and negative one for the lower limit. These coded designs were used in the initial lower W_i limits of zero.

Sampled MOE's and Research Design for Revised Limits:

CF	CV	AF	W ₁	W ₂	W ₃	W ₄
932.67	1190.02	1736.46	450	750	880	312
868.40	1159.59	1618.73	450	450	880	312
862.12	1170.66	1652.86	300	750	880	312
797.85	1139.65	1504.04	300	450	880	312
873.84	1165.56	1677.36	375	600	1040	424
868.13	1165.56	1645.10	375	600	1040	200
865.41	1164.70	1626.09	375	600	720	424
859.61	1164.31	1585.47	375	600	720	200
904.92	1174.87	1701.47	450	600	880	424
899.15	1174.81	1668.05	450	600	880	200
834.40	1155.39	1602.27	300	600	880	424
828.60	1155.32	1567.22	300	600	880	200
901.59	1180.83	1717.52	375	750	1040	312
893.14	1179.81	1673.61	375	750	720	312
837.39	1150.28	1594.91	375	450	1040	312
828.87	1148.42	1526.83	375	450	720	312
906.24	1175.30	1708.49	450	600	1040	312
897.79	1174.29	1657.15	450	600	720	312
835.76	1155.82	1612.06	300	600	1040	312
827.24	1153.96	1554.73	300	600	720	312
900.26	1180.40	1710.86	375	750	880	424
894.50	1180.34	1681.32	375	750	880	200
836.03	1149.85	1585.12	375	450	880	424
830.23	1149.79	1540.05	375	450	880	200
866.77	1165.13	1635.88	375	600	880	312

The next two examples are for coded CF and CV research designs and their sampled values. These designs were used in the initial lower limit of zero.

APPENDIX C

Regression

This section includes a complete stepwise regression for CF, and the final step and residual scatter diagrams for CV and AF regressions. It concludes with Tables VI, VII, and VIII which show the regression error for each surface.

The Statistical Package for the Social Sciences (SPSS) was employed with a stepwise regression option for the determination of the regression polynomials.

Example CF Stepwise Regression:

1
T

84/03/31. 15.56.17. PAGE 1

ASD COMPUTER CENTER
WRIGHT-PATTERSON AFB, OHIO

S P S S - - STATISTICAL PACKAGE FOR THE SOCIAL SCIENCES

VERSION 8.3 (NOS) -- MAY 04, 1982

376500 CM MAXIMUM FIELD LENGTH REQUEST

RUN NAME SECOND ORDER REGRESSION FOR 4 INDEP. VARIABLES
 FILE NAME REGRESSION ANALYSIS WITH RGDAT
 VARIABLE LIST CF,CV,AF,W1,W2,W3,W4
 N OF CASES 25
 INPUT MEDIUM CARD
 INPUT FORMAT FREEFIELD
 COMPUTE W11=W1**2
 COMPUTE W22=W2**2
 COMPUTE W33=W3**2
 COMPUTE W44=W4**2
 COMPUTE W12=W1*W2
 COMPUTE W13=W1*W3
 COMPUTE W14=W1*W4
 COMPUTE W23=W2*W3
 COMPUTE W24=W2*W4
 COMPUTE W34=W3*W4

CPU TIME REQUIRED.. .044 SECONDS

REGRESSION METHOD=STEPWISE/
 VARIABLES=CF TO W4,W11,W22,W33,W44,W12,W13,W14,W23,W24,W34
 REGRESSION=CF WITH W1 TO W34/RESIDUALS/
 REGRESSION=CV WITH W1 TO W34/RESIDUALS/
 REGRESSION=AF WITH W1 TO W34/RESIDUALS/
 STATISTICS ALL

00056000 CM NEEDED FOR REGRESSION

1SECOND ORDER REGRESSION FOR 4 INDEP. VAR 84/03/31. 15.56.17. PAGE 2

FILE - REGRESSI (CREATED - 84/03/31) ANALYSIS WITH RGDAT

***** MULTIPLE REGRESSION *****

VARIABLE	MEAN	STANDARD DEV	CASES
CF	866.0364	33.9360	25
CV	1164.9860	12.9165	25
AF	1631.3460	63.6308	25
W1	375.0000	53.0330	25
W2	600.0000	106.0660	25
W3	880.0000	113.1371	25
W4	312.0000	79.1960	25
W11	143325.0000	39878.0367	25
W22	370800.0000	127795.2464	25
W33	786688.0000	199548.6748	25

W44	103365.1200	49830.4904	25
W12	225000.0000	51143.1814	25
W13	330000.0000	63261.3626	25
W14	117000.0000	34169.2845	25
W23	528000.0000	115827.4579	25
W24	187200.0000	58310.2049	25
W34	274560.0000	78463.7564	25

CORRELATION COEFFICIENTS.

A VALUE OF 99.00000 IS PRINTED
IF A COEFFICIENT CANNOT BE COMPUTED.

CV	.96255					
AF	.94427	.93601				
W1	.73483	.53064	.55301			
W2	.66939	.84161	.74357	0		
W3	.08836	.03570	.30704	0	0	
W4	.06015	.00292	.19999	0	0	0
W11	.73340	.53717	.55226	.99741	0	0
W22	.66466	.83810	.73667	0	.99596	0
W33	.08858	.03541	.30665	0	0	.99786
W44	.06047	.00376	.20068	.00000	.00000	.00000
W12	.97779	.98924	.91798	.62217	.77771	0
W13	.60135	.42093	.61314	.73772	0	.67065
W14	.40811	.26336	.44135	.48424	0	0
W23	.59119	.69868	.77587	0	.80584	.58606
W24	.42890	.48001	.58204	0	.56753	0
W34	.09315	.01846	.31451	0	0	.44987

	CF	CV	AF	W1	W2	W3
W11	0					
W22	0	-.00182				
W33	0	-.00133	-.00166			
W44	.99173	-.00260	-.00325	-.00237		
W12	0	.62056	.77457	0	0	
W13	0	.73581	0	.66922	0	.45899
W14	.86916	.48299	0	0	.86197	.30128
W23	0	0	.80258	.58481	0	.62671
W24	.81491	0	.56524	0	.80817	.44137
W34	.88821	0	0	.44891	.88086	0

W4	W11	W22	W33	W44	W12
----	-----	-----	-----	-----	-----

W14	.35724			
W23	.39305	0		
W24	0	.70829	.45733	
W34	.30171	.77200	.26365	.72381

W13	W14	W23	W24
-----	-----	-----	-----

1SECOND ORDER REGRESSION FOR 4 INDEP. VAR 84/03/31. 15.56.17. PAGE 3

FILE - REGRESSI (CREATED - 84/03/31) ANALYSIS WITH RGDAT

***** MULTIPLE REGRESSION *****

DEP. VAR... CF

MEAN RESPONSE 366.03640 STD. DEV. 33.93598

VARIABLE(S) ENTERED ON STEP 1
W12

MULTIPLE R	.9778	ANOVA	DF	SUM SQUARES	MEAN SQ.	F
R SQUARE	.9561	REGRESSION	1.	26425.270	26425.270	500.498
STD DEV	7.2662	RESIDUAL	23.	1214.352	52.798	SIG. 0
ADJ R SQUARE	.9542	COEFF OF VARIABILITY		.8PCT		

VARIABLE	B	S.E. B	F	SIG.	BETA	ELASTICITY
W12	.001	.000	500.498	0	.97779	.16856
CONSTANT	720.055	6.685	11601.442	0		

VARIABLE(S) ENTERED ON STEP 2
W13

MULTIPLE R	.9927	ANOVA	DF	SUM SQUARES	MEAN SQ.	F
R SQUARE	.9855	REGRESSION	2.	27240.206	13620.103	750.201
STD DEV	4.2609	RESIDUAL	22.	399.416	18.155	SIG. .000
ADJ R SQUARE	.9842	COEFF OF VARIABILITY		.5PCT		

VARIABLE	B	S.E. B	F	SIG.	BETA	ELASTICITY
W12	.001	.000	949.877	0	.88908	.15327
W13	.000	.000	44.887	.000	.19327	.03951
CONSTANT	699.085	5.016	19421.403	0		

1SECOND ORDER REGRESSION FOR 4 INDEP. VAR 84/03/31. 15.56.17. PAGE 4

FILE - REGRESSI (CREATED - 84/03/31) ANALYSIS WITH RGDAT

W23	-.000	.000	7.607	.016	-.00249	-.00044
W33	-.000	.000	7.593	.017	-.00360	-.00056
W24	-.000	.000	2.349	.136	-.00102	-.00013
W44	-.000	.000	1.479	.247	-.00081	-.00007
CONSTANT	504.445	.360	.196E+07	0		

1SECOND ORDER REGRESSION FOR 4 INDEP. VAR 84/03/31. 15.56.17. PAGE 12

FILE - REGRESSI (CREATED - 84/03/31) ANALYSIS WITH REGDAT

***** MULTIPLE REGRESSION *****

DEP. VAR... CF

VARIABLE(S) ENTERED ON STEP 17

W11

MULTIPLE R	1.0000	ANOVA	DF	SUM SQUARES	MEAN SQ.	F
R SQUARE	1.0000	REGRESSION	13.	27639.620	2126.125	.125E+08
STD DEV	.0131	RESIDUAL	11.	.002	.000	SIG. 0
ADJ R SQUARE	1.0000	COEFF OF VARIABILITY		.0PCT		

VARIABLE	B	S.E. B	F	SIG.	BETA	ELASTICITY
W13	-.000	.000	7.186	.021	-.00272	-.00056
W14	-.000	.000	1.320	.275	-.00090	-.00012
W2	.296	.000	.357E+06	0	.92361	.20473
W1	.472	.001	.163E+06	0	.73693	.20419
W22	-.000	.000	37746.079	0	-.25266	-.02873
W3	.029	.001	2272.737	0	.09629	.02935
W4	.028	.001	1977.203	0	.06502	.01004
W34	-.000	.000	11.879	.005	-.00290	-.00040
W23	-.000	.000	7.186	.021	-.00249	-.00044
W33	-.000	.000	3.523	.087	-.00335	-.00052
W24	-.000	.000	2.347	.154	-.00102	-.00013
W44	-.000	.000	.564	.469	-.00068	-.00006
W11	.000	.000	.046	.834	.00035	.00005
CONSTANT	504.542	.590	.732E+06	0		

F-LEVEL OR TOLERANCE-LEVEL INSUFFICIENT FOR FURTHER COMPUTATION.

COEFFICIENTS AND CONFIDENCE INTERVALS.

VARIABLE	B	95 PCT C.I.	
W13	-.0000	-.0000	-.0000
W14	-.0000	-.0000	.0000
W2	.2955	.2944	.2966
W1	.4716	.4690	.4741
W22	-.0001	-.0001	-.0001
W3	.0289	.0276	.0302
W4	.0279	.0265	.0292
W34	-.0000	-.0000	-.0000
W23	-.0000	-.0000	-.0000
W33	-.0000	-.0000	.0000
W24	-.0000	-.0000	.0000
W44	-.0000	-.0000	.0000
W11	.0000	-.0000	.0000
CONSTANT	504.5424	503.2446	505.8401

1SECOND ORDER REGRESSION FOR 4 INDEP. VAR 84/03/31. 15.56.17. PAGE 13

FILE - REGRESS1 (CREATED - 84/03/31) ANALYSIS WITH RGDAT

***** MULTIPLE REGRESSION *****

DEP. VAR... CF

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

W1	.00000					
W2	.00000	.00000				
W3	.00000	.00000	.00000			
W4	.00000	.00000	.00000	.00000		
W11	-.00000	-.00000	-.00000	-.00000	.00000	
W22	-.00000	-.00000	-.00000	-.00000	.00000	.00000
W33	-.00000	-.00000	-.00000	-.00000	.00000	.00000
W44	-.00000	-.00000	-.00000	-.00000	.00000	.00000
W13	-.00000	-.00000	-.00000	-.00000	.00000	.00000
W14	-.00000	-.00000	-.00000	-.00000	.00000	.00000
W23	-.00000	-.00000	-.00000	-.00000	.00000	.00000
W24	-.00000	-.00000	-.00000	-.00000	.00000	.00000
W34	-.00000	-.00000	-.00000	-.00000	.00000	.00000

W1	W2	W3	W4	W11	W22
----	----	----	----	-----	-----

W33	.00000					
W44	.00000	.00000				
W13	.00000	.00000	.00000			
W14	.00000	.00000	-.00000	.00000		
W23	.00000	.00000	-.00000	-.00000	.00000	
W24	.00000	.00000	-.00000	.00000	-.00000	.00000
W34	.00000	.00000	-.00000	-.00000	-.00000	.00000

W33	W44	W13	W14	W23	W24
-----	-----	-----	-----	-----	-----

W34 .00000

W34

1SECOND ORDER REGRESSION FOR 4 INDEP. VAR 84/03/31. 15.56.17. PAGE 14

FILE - REGRESSI (CREATED - 84/03/31) ANALYSIS WITH RGDAT

***** MULTIPLE REGRESSION *****

DEP. VAR... CF

SUMMARY TABLE.

STEP	VARIABLE	E/R	F	MULT-R	R-SQ	CHANGE	R	OVERALL F	SIG.
1	W12	E	500.498	.978	.956	.956	.978	500.498	0
2	W13	E	44.887	.993	.986	.029	.601	750.201	.000
3	W14	E	14.803	.996	.992	.006	.408	618.861	0
4	W11	E	5.995	.997	.993	.002	.733	761.719	.000
5	W2	E	53.057	.999	.998	.005	.669	2206.100	0
6	W1	E	31.552	1.000	.999	.001	.735	4799.883	.000
7	W12	R	.000	1.000	.999	0	.978	6079.851	0
8	W22	E	54.029	1.000	1.000	.000	.665	19216.179	.000
9	W11	R	.002	1.000	1.000	-.000	.733	24337.452	0
10	W3	E	39.242	1.000	1.000	.000	.088	61108.470	.000
11	W4	E	3411.581	1.000	1.000	.000	.060	.998E+07	0
12	W34	E	6.890	1.000	1.000	.000	.093	.118E+08	.000
13	W23	E	5.284	1.000	1.000	.000	.591	.132E+08	0
14	W33	E	5.359	1.000	1.000	.000	.089	.154E+08	.000
15	W24	E	2.459	1.000	1.000	.000	.429	.154E+08	0
16	W44	E	1.479	1.000	1.000	.000	.060	.147E+08	.000
17	W11	E	.046	1.000	1.000	.000	.733	.125E+08	0

1SECOND ORDER REGRESSION FOR 4 INDEP. VAR 84/03/31. 15.56.17. PAGE 15

FILE - REGRESSI (CREATED - 84/03/31) ANALYSIS WITH RGDAT

***** MULTIPLE REGRESSION *****

RESIDUAL PLOT.

Y VALUE	Y EST.	RESIDUAL	-2SD	0.0	+2SD
932.670	932.655	.015		I	
868.400	868.403	-.003		I	
862.120	862.121	-.001		I	
797.850	797.870	-.020		I	
873.840	873.855	-.015		I	
868.130	868.126	.004		I	
865.410	865.418	-.008		I	
859.610	859.600	.010		I	
904.920	904.912	.008		I	
899.150	899.153	-.003		I	
834.400	834.393	.007		I	
828.600	828.605	-.005		I	
901.590	901.595	-.005		I	
893.140	893.148	-.008		I	
837.390	837.378	.012		I	
828.870	828.862	.008		I	
906.240	906.247	-.007		I	
897.790	897.800	-.010		I	
835.760	835.749	.011		I	
827.240	827.232	.008		I	
900.260	900.257	.003		I	
894.500	894.504	-.004		I	
836.030	836.025	.005		I	
830.230	830.232	-.002		I	
866.770	866.770	.000			

NOTE - (*) INDICATES ESTIMATE CALCULATED WITH MEANS SUBSTITUTED
R INDICATES POINT OUT OF RANGE OF PLOT

NUMBER OF CASES PLOTTED	25.	
NUMBER OF 2 S.D. OUTLIERS	0 OR	0 PERCENT OF THE TOTAL
VON NEUMANN RATIO	1.79118	DURBIN-WATSON TEST 1.71954
NUMBER OF POSITIVE RESIDUALS	12.	
NUMBER OF NEGATIVE RESIDUALS	13.	
NUMBER OF RUNS OF SIGNS	15.	
EXPECTED NUMBER OF RUNS OF SIGNS	13.	
EXPECTED S.D. OF RUN DISTRIBUTION	2.44328	
UNIT NORMAL DEViate-		
$Z = (\text{OBSERVED} - \text{EXPECTED} + 0.5) / \text{S.D.}$.82676
PROBABILITY OF OBTAINING .GE. ABS(Z)		.20419

CF Summary Regression:

***** MULTIPLE REGRESSION *****

DEP. VAR... CV

VARIABLE(S) ENTERED ON STEP 12
W4

MULTIPLE R	1.0000	ANOVA	DF	SUM SQUARES	MEAN SQ.	F
R SQUARE	.9999	REGRESSION	12.	4003.748	333.646	13552.390
STD DEV	.1569	RESIDUAL	12.	.295	.025	SIG. .000
ADJ R SQUARE	.9999	COEFF OF VARIABILITY		.0PCT		

VARIABLE	B	S.E. B	F	SIG.	BETA	ELASTICITY
W12	-.000	.000	3.416	.089	-.05103	-.00249
W2	.120	.006	358.895	.000	.93394	.06171
W1	.166	.014	133.499	.000	.68084	.05338
W3	.031	.007	17.845	.001	.26941	.02323
W13	-.000	.000	7.165	.020	-.08571	-.00496
W23	-.000	.000	7.165	.020	-.07846	-.00397
W33	-.000	.000	4.195	.063	-.11542	-.00504
W44	.000	.000	.523	.484	.02076	.00048
W34	-.000	.000	1.545	.238	-.03305	-.00128
W22	-.000	.000	.929	.354	-.03958	-.00127
W11	-.000	.000	.853	.374	-.04734	-.00189
W4	.002	.006	.099	.758	.01169	.00051
CONSTANT	1027.039	7.121	20799.923	0		

F-LEVEL OR TOLERANCE-LEVEL INSUFFICIENT FOR FURTHER COMPUTATION.

COEFFICIENTS AND CONFIDENCE INTERVALS.

VARIABLE	B	95 PCT C.I.	
W12	-.0000	-.0000	.0000
W2	.1198	.1060	.1336
W1	.1658	.1346	.1971
W3	.0308	.0149	.0466
W13	-.0000	-.0000	-.0000
W23	-.0000	-.0000	-.0000
W33	-.0000	-.0000	.0000
W44	.0000	-.0000	.0000
W34	-.0000	-.0000	.0000
W22	-.0000	-.0000	.0000
W11	-.0000	-.0001	.0000
W4	.0019	-.0113	.0151
CONSTANT	.10E+04	.10E+04	.10E+04

RESIDUAL PLOT.

Y VALUE	Y EST.	RESIDUAL	-2SD	0.0	+2SD
1190.020	1190.021	-.001		.	
1159.590	1159.565	.025		I .	
1170.660	1170.633	.027		I .	
1139.650	1139.596	.054		I .	
1165.560	1165.618	-.058		I .	
1165.560	1165.706	-.146		I .	
1164.700	1164.501	.199		I .	
1164.310	1164.200	.110		I .	
1174.870	1175.004	-.134		I .	
1174.810	1174.897	-.087		I .	
1155.390	1155.325	.065		I .	
1155.320	1155.219	.101		I .	
1180.830	1180.668	.162		I .	
1179.810	1179.776	.034		I .	
1150.280	1150.341	-.061		I .	
1148.420	1148.610	-.190		I .	
1175.300	1175.137	.163		I .	
1174.280	1174.246	.034		I .	
1155.820	1155.879	-.059		I .	
1153.960	1154.147	-.187		I .	
1180.400	1180.534	-.134		I .	
1180.340	1180.428	-.088		I .	
1149.850	1149.788	.062		I .	
1149.790	1149.681	.109		I .	
1165.130	1165.130	.000		.	

NOTE - (*) INDICATES ESTIMATE CALCULATED WITH MEANS SUBSTITUTED
R INDICATES POINT OUT OF RANGE OF PLOT

NUMBER OF CASES PLOTTED 25.
NUMBER OF 2 S.D. OUTLIERS 0 OR 0 PERCENT OF THE TOTAL
VON NEUMANN RATIO 1.71919 DURBIN-WATSON TEST 1.85042

AF Summary Regression:

DEP. VAR... AF

VARIABLE(S) ENTERED ON STEP 14

W44

MULTIPLE R	.9999	ANOVA	DF	SUM SQUARES	MEAN SQ.	F
R SQUARE	.9999	REGRESSION	14.	97163.086	6940.220	6849.74
STD DEV	1.0066	RESIDUAL	10.	10.132	1.013	SIG. .00
ADJ R SQUARE	.9997	COEFF OF VARIABILITY		.1PCT		

VARIABLE	B	S.E. B	F	SIG.	BETA	ELASTICITY
W12	-.001	.000	238.497	.000	-.55530	-.09529
W34	-.000	.000	17.245	.002	-.14382	-.01963
W3	.523	.047	125.163	.000	.28190	
W2	1.335	.042	1028.334	.000	2.22576	.49111
W1	1.357	.094	208.661	.000	1.13109	.31196
W22	-.000	.000	110.898	.000	-.56309	-.06373
W23	-.000	.000	144.143	.000	-.45830	-.08149
W4	.435	.048	81.119	.000	.54150	.08321
W24	-.000	.000	59.509	.000	-.21173	-.02652
W13	-.000	.000	8.853	.014	-.12407	-.02524
W33	-.000	.000	7.885	.019	-.20606	-.03169
W11	-.000	.000	3.723	.083	-.12878	-.01805
W14	-.0000	.656	.437	-.02605	-.00348	
W44	-.000	.000	.250	.628	-.01870	-.00151
CONSTANT	323.748	46.559	48.351	.000		

ALL VARIABLES ARE IN THE EQUATION.

COEFFICIENTS AND CONFIDENCE INTERVALS.

VARIABLE	B	95 PCT C.I.
W12	-.0007	-.0008 -.0006
W34	-.0001	-.0002 -.0001
W3	.5226	.4185 .6267
W2	1.3353	1.2425 1.4281
W1	1.3571	1.1478 1.5665
W22	-.0003	-.0003 -.0002
W23	-.0003	-.0003 -.0002
W4	.4351	.5427
W24	-.0002	-.0003 -.0002
W13	-.0001	-.0002 -.0000
W33	-.0001	-.0001 -.0000
W11	-.0002	-.0004 .0000
W14	-.0000	-.0002 .0001
W44	-.0000	-.0001 .0001
CONSTANT	323.7475	220.0071 427.4879

1 SECOND ORDER REGRESSION FOR 4 INDEP. VAR 04/03/81 10:58:17 PAGE

TABLE VI
CF Regression Error

Design Points						
W_1	W_2	W_3	W_4	CF_m	CF_e	Residual
450	750	880	312	932.67	932.655	.015
450	450	880	312	868.40	868.403	.003
300	750	880	312	862.12	862.121	.001
300	450	880	312	797.85	797.870	.020
375	600	1040	424	873.84	873.855	.015
375	600	1040	200	868.13	868.126	.004
375	600	720	424	865.41	865.418	.008
375	600	720	200	859.61	859.600	.010
450	600	880	424	904.92	904.912	.008
450	600	880	200	899.15	899.153	.003
300	600	880	424	834.40	834.393	.007
300	600	880	200	828.60	828.605	.005
375	750	1040	312	901.59	901.595	.005
375	750	720	312	893.14	893.148	.008
375	450	1040	312	837.39	837.378	.012
375	450	720	312	828.87	828.862	.008
450	600	1040	312	906.24	906.247	.007
450	600	720	312	897.79	897.800	.010
300	600	1040	312	835.76	835.749	.011
300	600	720	312	827.24	827.232	.008
375	750	880	424	900.26	900.257	.003
375	750	880	200	894.50	894.504	.004
375	450	880	424	836.03	836.025	.005
375	450	880	200	830.23	830.232	.002
375	600	880	312	866.77	866.770	.000
Design Points Standard Deviation of Error = 0.0131						

continued

Table VI continued

Random Sample						
441	469	803	226	864.41	864.289	.121
328	458	752	286	808.84	808.915	.075
391	604	923	333	876.80	876.780	.021
300	645	969	260	841.71	841.990	.280
423	711	853	357	912.46	912.689	.228
350	480	825	275	826.08	825.885	.195
429	577	726	208	880.68	880.385	.294
403	585	789	241	872.62	872.423	.196
408	470	947	410	857.71	857.549	.160
489	545	791	216	904.31	903.527	.783
355	462	894	392	829.02	828.921	.099
Random Sample Mean Error, $\bar{e} = 0.223$						
Random Sample Standard Deviation of Error = 0.204						

TABLE VII
CV Regression Error

Design Points						
W_1	W_2	W_3	W_4	CV_m	CV_e	Residual
450	750	880	312	1190.02	1190.021	.001
450	450	880	312	1159.59	1159.565	.025
300	750	880	312	1170.66	1170.633	.027
300	450	880	312	1139.65	1139.596	.054
375	600	1040	424	1165.56	1165.618	.058
375	600	1040	200	1165.56	1165.706	.146
375	600	720	424	1164.70	1164.501	.199
375	600	720	200	1164.31	1164.200	.110
450	600	880	424	1174.87	1175.004	.134
450	600	880	200	1174.81	1174.897	.087
300	600	880	424	1155.39	1155.325	.065
300	600	880	200	1155.32	1155.219	.101
375	750	1040	312	1180.83	1180.668	.162
375	750	720	312	1179.81	1179.776	.034
375	450	1040	312	1150.28	1150.341	.061
375	450	720	312	1148.42	1148.610	.190
450	600	1040	312	1175.30	1175.137	.163
450	600	720	312	1174.28	1174.246	.034
300	600	1040	312	1155.82	1155.879	.059
300	600	720	312	1153.96	1154.147	.187
375	750	880	424	1180.40	1180.534	.134
375	750	880	200	1180.34	1180.428	.088
375	450	880	424	1149.85	1149.788	.062
375	450	880	200	1149.79	1149.681	.109
375	600	880	312	1165.13	1165.130	.000
Design Point Standard Deviation of Error = 0.1569						

continued

Table VII continued

Random Sample						
441	469	803	226	1159.99	1159.955	.035
328	458	752	286	1143.91	1143.285	.625
391	604	923	333	1167.73	1167.811	.081
300	645	969	260	1160.21	1160.778	.568
423	711	853	357	1182.59	1182.649	.059
350	480	825	275	1149.49	1149.118	.372
429	577	726	208	1169.02	1169.070	.050
403	585	789	241	1166.83	1167.267	.437
408	470	947	410	1156.36	1156.467	.107
489	545	791	216	1173.88	1174.003	.123
355	462	894	392	1148.52	1148.406	.114
Random Sample Mean Error, $\bar{e} = 0.234$						
Random Sample Standard Deviation of Error = 0.222						

TABLE VIII
AF Regression Error

Design Points						
W_1	W_2	W_3	W_4	AF_m	AF_e	Residual
450	750	880	312	1736.46	1737.320	.860
450	450	880	312	1618.72	1619.040	.310
300	750	880	312	1652.86	1653.337	.477
300	450	880	312	1504.04	1503.967	.073
375	600	1040	424	1677.36	1677.435	.075
375	600	1040	200	1645.10	1645.622	.522
375	600	720	424	1626.09	1626.355	.265
375	600	720	200	1585.47	1586.182	.712
450	600	880	424	1701.47	1701.778	.308
450	600	880	200	1668.05	1666.600	1.450
300	600	880	424	1602.27	1603.065	.795
300	600	880	200	1567.22	1566.256	.964
375	750	1040	312	1717.52	1716.390	1.130
375	750	720	312	1673.61	1673.215	.395
375	450	1040	312	1594.91	1594.650	.260
375	450	720	312	1526.83	1527.305	.475
450	600	1040	312	1708.49	1708.939	.449
450	600	720	312	1657.15	1656.674	.476
300	600	1040	312	1612.06	1612.405	.345
300	600	720	312	1554.73	1554.150	.588
375	750	880	424	1710.86	1710.299	.581
375	750	880	200	1681.32	1682.070	.750
375	450	880	424	1585.12	1584.239	.881
375	450	880	200	1540.05	1540.480	.430
375	600	880	312	1635.88	1635.880	.000
Design Points Standard Deviation of Error = 1.0066						

continued

Table VIII continued

Random Sample						
441	469	803	226	1591.84	1588.798	3.042
328	458	752	286	1494.51	1496.747	2.237
391	604	923	333	1658.73	1658.556	.170
300	645	969	260	1612.72	1613.919	.199
423	711	853	357	1711.99	1711.877	.113
350	480	825	275	1541.90	1540.694	1.206
429	577	726	208	1614.18	1614.414	.234
403	585	789	241	1619.45	1619.095	.355
408	470	947	410	1626.35	1628.740	2.390
489	545	791	216	1654.74	1654.590	.150
355	462	894	392	1571.61	1571.837	2.770
Random Sample Mean Error, $\bar{e} = 1.233$						
Random Sample Standard Deviation of Error = 1.154						

APPENDIX D

Sequential Unconstrained Minimization Technique

(SUMT)

The SUMT program was developed by Mylander, et al (23) based on the nonlinear programming work of Fiacco and McCormick (12). A descriptive pamphlet of the program was developed by Captain R. M. Floyd for the Air Force Institute of Technology. With his permission, this pamphlet is reproduced in this appendix. Following the pamphlet is an example of both an executive program that was used in the study and a sample output.

DESCRIPTION*
OF
SUMT LIBRARY

CAPT R.M. FLOYD
FEB 1982

*FORTRAN program SUMT code and description adapted from
References (2) and (3).

SUMT LIBRARY

A. Purpose

This program finds the minimum of a multivariable, nonlinear function subject to nonlinear inequality and equality constraints:

$$\text{Minimize} \quad F(X_1, X_2, \dots, X_N)$$

$$\text{Subject to} \quad G_k(X_1, X_2, \dots, X_N) \geq 0, \quad k = 1, 2, \dots, M$$

$$H_k(X_1, X_2, \dots, X_N) = 0, \quad k = M+1, M+2, \dots, M+MZ$$

B. Method

The procedure was developed by Fiacco and McCormick (1). The technique uses the problem constraints and the original objective function to form an unconstrained objective function which is minimized by any appropriate unconstrained, multivariable technique.

The algorithm proceeds as follows:

- 1) A modified objective function is formulated consisting of the original function and penalty functions with the form

$$P = F - r \sum_{k=1}^M \ln G_k + \sum_{k=M+1}^{M+MZ} H_k^2/r$$

where r is a positive constant. As the algorithm progresses, r is reevaluated to form a monotonically decreasing sequence $r_1 > r_2 > \dots > 0$. As r becomes small, under suitable conditions P approaches F and the problem is solved.

- 2) Select a starting point (feasible or nonfeasible) and an initial value for r .
 - 3) Determine the minimum of the modified objective function for the current value of r using an appropriate technique (several options available).
-

- 4) Estimate the optimal solution using extrapolation formulas (1).
- 5) Select a new value for r (1) and repeat the procedure until the convergence criterion is satisfied.

A logic diagram for this method is given in Figure 1.

C. Use of the SUMT Library

Execution of the SUMT routines requires that the user submit a computer job composed of the following elements: 1) Job Control commands, 2) a MAIN program, 3) user-supplied sub-routines, and 4) NAMELIST data. The layout of these elements is shown below.

```
JOB CARDS
(7/8/9)
MAIN PROGRAM
USER-SUPPLIED SUBROUTINES
(7/8/9)
NAMELIST DATA
(6/7/8/9)
```

Note that an end-of-record and an end-of-file is signified by a multipunch (7/8/9) and (6/7/8/9), respectively in column 1 of a keypunch card (or by *EOR and *EOF, respectively when coded at a computer terminal). Each element is described separately below. In Section E an example is presented which combines these elements for job execution.

1) Job Cards

The Job Card sequence required to execute SUMT is

```
abc,Tt,Iok,CM100000. PN,name
ATTACH,lfn,SUMT,ID=T800535.
LIBRARY,lfn.
FTN.
LGO.
```

in which

```
abc  : 3 character identifier
t    : decimal seconds of Central Processor time
      requested for the job (default is 20)
k    : decimal seconds of Input/Output time requested
      for the job (default is 70)
PN   : user computer problem number
name : user name or other identifying information
lfn  : arbitrary local file name for the SUMT library
```

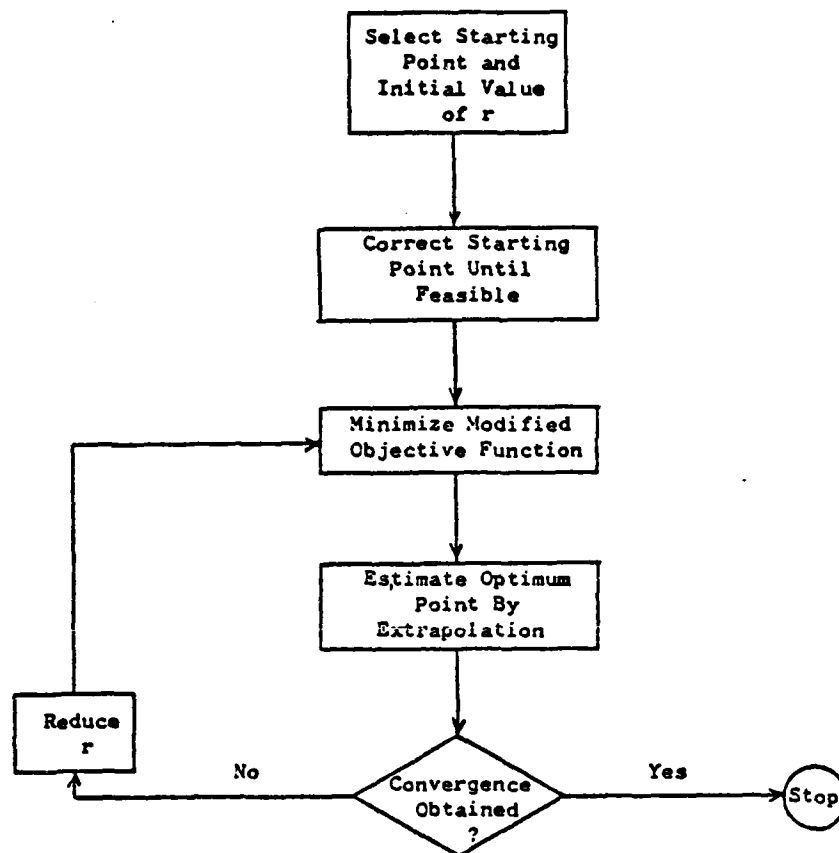


Figure 1. Fiacco and McCormick (SLMT ALGORITHM)
Logic Diagram

2) MAIN Program

The MAIN program identifies three files and assigns corresponding device numbers in a Program statement. It communicates the device numbers through the labelled COMMON:

```
COMMON/DEVC/NI,NO,NS
```

The variables NI, NO, NS are given values by MAIN and correspond to the device numbers for the input and output files and a save file, respectively. Execution of the SUMT routines is initiated by a call to subroutine SUMT. The code given below demonstrates the requirements of the MAIN routine:

```
PROGRAM MAIN(INPUT,OUTPUT,SAVE,TAPE5=INPUT,  
  TAPE6=OUTPUT,TAPE7=SAVE)  
COMMON/DEVC/NI,NO,NS  
NI=5  
NO=6  
NS=7  
CALL SUMT  
STOP  
END
```

3) User-Supplied Subroutines

The subroutines described below are called by the given names and with the given argument lists. Inclusion of the routine RESTNT is mandatory, while routines GRAD1 and MATRIX are optional (default routines are coded in SUMT). Each user-supplied subroutine must contain the COMMON card:

```
COMMON/SHARE/X(100),DEL(100),A(100,100),N(5)
```

This common region is used for communication between the SUMT routines and the user-supplied routines. For each routine, the argument IN is an input argument referring to the objective function, the inequality constraints, or the equality constraints as follow:

IN = 0	: objective function is being considered
IN = 1,2,...,M	: the INth inequality constraint is being considered
IN = M+1,M+2,...,M+M2	: the 1st, 2nd,..., M2th equality constraint is being considered

RESTNT(IN,VAL)

This subroutine sets the argument VAL to the value of the objective function, an inequality constraint, or an equality constraint according to the value of the argument IN. The current values of the independent variables are found in the vector X of the labelled COMMON/SHARE/.

GRAD1(IN)

This subroutine computes the gradient of the objective function, an inequality constraint, or an equality constraint according to the value of the argument IN. The gradient with respect to the Ith independent variable is stored in location DEL(I). The current values of the independent variables are in the vector X, and both X and DEL are in labelled COMMON/SHARE/. The elements of DEL are not set to zero prior to entry into GRAD1. The routine GRAD1 included by default in SUMT computes a numerical approximation for each gradient using the central difference technique. If the user wishes to provide some of the gradients analytically and others by numerical differencing, the central difference approximation for any desired gradient as indexed by the argument IN may be computed using a call to the SUMT library routine DIFP1 as

CALL DIFP1(IN)

with selective execution according to the value of IN. In the usual case, user's rely on the default GRAD1 routine rather than provide a replacement for it with analytically determined gradients.

MATRIX(IN,L)

This subroutine computes the upper triangle and diagonal elements of the matrix of second partial derivatives of the objective function, an inequality constraint, or an equality constraint according to the value of the argument IN. The current values of the independent variables are in the vector X, and the computed second partials are stored in array A. Both X and A are in the labelled COMMON/SHARE/. The elements

of the array A below the main diagonal must not be modified within MATRIX; the upper triangle of A is set to zero before entry into MATRIX. The second argument (L) is provided so that the user may inform SUMT that the matrix of second partial derivatives of a function is zero: L should be set to 1 according to the value of argument IN if the corresponding matrix of second partials is zero. The routine MATRIX included by default in SUMT computes a numerical approximation for each matrix of second partial derivatives using central differences to second order. If the user wishes to provide some of the matrices analytically and others by numerical differencing, the central difference for any desired matrix of second partials as indexed by the argument IN may be computed using a call to the SUMT library routine DIFF2 as

```
CALL DIFF2(IN)
```

with selective execution according to the value of IN. In the usual case, user's rely on the default MATRIX routine rather than provide a replacement for it with analytically determined second partial derivative matrices.

4) NAMelist Data

Data is entered into SUMT using a NAMELIST read from the input file; the NAMELIST is referred to as DATA and includes the following variables:

```
NAMelist/DATA/EPsi,RHoin,THETA0,RATIO,TMMAX,M,N,MZ,  
X,NT,XEP1,XEP2,NEXOP1,NEXOP2
```

The variables X and NT are vectors of length 100 and 10, respectively. All of the variables in the NAMELIST are assigned default values by SUMT (see Section 4b below). To assign different values to any subset of these variables an assignment is made to each appropriate variable name in the NAMELIST input. The complete data set is delimited on the left by \$DATA and on the right by \$END; individual data assignments are separated by commas; the data set may extend

over one or more 80 column fields. No information should be placed in column 1 of any data field, since the NAMELIST read processes information beginning with column 2. Refer to an appropriate FORTRAN manual for more information on NAMELIST input.

a. Definition of NAMELIST Data Items

- EPSI Tolerance, ϵ , used to decide if an unconstrained minimum has been achieved for each subproblem (see NT(7))
- RHOIN Possible initial value of r , r_1 , (often set at 1.0) (see NT(1))
- THETA0 Tolerance, θ_0 , used to decide if the solution to the problem has been approximated (see NT(5))
- RATIO Parameter, C , (>1) used to compute consecutive values of r ; $r_{i+1} = r_i/C$ (often set at 16.0)
- DTMAX Maximum amount of time for solving problem (sec.)
- M Number of inequality constraints
- N Number of independent variables
- MZ Number of equality constraints
- X Independent variables (starting point on input)
- NT(1) Option key for r values, as follows:
- =1 The value for r is made by finding an approximation solution $\min\{[\nabla P(X^0, r)][\nabla^2 P(X^0, r)]^{-1}\nabla P(X^0, r)\}$ which is a good approximation only when X^0 is close to the boundary (i.e., for some i , $G_i(X)$ is close to zero) or when $\nabla^2 F(X^0)=0$ and when $MZ=0$
 - =2 r_1 is given by formula 8.65 [Ref. (1) p. 191 .] (Only can be used when $MZ=0$)
 - =3 $r_1 = \text{RHOIN}$
(normally set to 3)
- NT(2) Option key for constraints as follows:
- =1 The requirements (trivial constraints) that $X_i \geq 0$ for $i = 1, \dots, N$ are to be automatically included in the problem
 - =2 The only constraints on the problem are those inputted by the user
- NT(3) Option key for printout as follows:
- =1 Standard printout (this includes a call to OUTPUT after the solution of every subproblem. Also the estimates of the "Lagrange multipliers" and first- and second-order solution estimates are printed)
 - =2 For additional printout (includes standard printout and every intermediate point, gradient of P and the vector S)
(normally set to 1)
- NT(4) Option key on final convergence as follows:
- =1 Final convergence is determined on the basis of current solution to the subproblem

- =2 Final convergence is determined on the basis of the first order estimates. The first order estimate of the solution vector must be close to feasible. See below.
- =3 Final convergence is determined on the basis of the second order estimates. The second order estimate of the solution vector must be close to feasible before the convergence check is made. If \underline{X} is a solution estimate it is considered close to being feasible if $G_i(\underline{X}) + \theta_0 \geq 0$, $i = 1, 2, \dots, M$.
(normally set to 1)

NT(5) Option key on final convergence as follows:
The convergence criterion determining the problem has been solved (Only use = 1, when NT(4) \neq 1)

- =1 Quit when $\frac{G - F[\underline{X}(r_k)]}{G[\underline{X}(r_k), \mu(r_k), \lambda(r_k)]} < \theta_0$
- =2 Quit when $|\sum_{j=1}^m \lambda_j G_j[\underline{X}(r_k)]| < \theta_0$ (use only for $M \neq 0$)
- =3 Quit when $\frac{\text{first order estimate of } v_0}{G[\underline{X}(r_k), \mu(r_k), \lambda(r_k)]} - 1 < \theta_0$
(normally set to 1)

NT(6) Option key for extrapolation as follows:

- =1 No extrapolation
- =2 Extrapolate through last two minima
- =3 Extrapolate through last three minima
(normally set to 1)

NT(7) Option key for subproblem convergence (fixed value of r) as follows:

- =1 Quit when $|\nabla_X^t P(X^i, r) \left[\frac{\partial^2 P(X, r)}{\partial X_i \partial X_j} \right]^{-1} \nabla_X P(X^i, r)| < \epsilon$
- =2 Quit when $|\nabla_X^t P(X^i, r) \left[\frac{\partial^2 P(X, r)}{\partial X_i \partial X_j} \right]^{-1} \nabla_X P(X^i, r)| < \frac{P(X^{i-1}) - P(X^i)}{5}$
- =3 Quit when $|\nabla_X P(X^i, r)| < \epsilon$

NT(8) Option key on problem linearity as follows:

- =1 At least one nonlinear constraint
- =2 Linear constraints
- =3 Linear constraints and linear objective function
(i.e., a linear programming problem)

When Option 3 = 3 MATRIX (the user subroutine supplying the second partial derivatives) will not be called, and when Option 3 = 2 it will be called only to get the second partials of $F(X)$.

NT(9) not used

NT(10) not used

- XEP1 Finite difference parameter used in DIFF1 if numerical derivatives used. Usually XEP1 = 0.0001 is satisfactory.
- XEP2 Iteration improvement limit. When minimizing the P-function for a given value of r (RHO) the value of P must decrease by an amount exceeding XEP2 for each iteration after the first. If it does not, then the code prints out the message "apparently roundoff errors prevent a more accurate determination of the minimum of this subproblem," and it is assumed a minimum has been found. (Normally, set XEP2 equal to 0.)
- NEXOP1 Key for checking derivatives as follows:
- =1 Solve problems without checking derivatives
 - =2 Solve problem after checking only first derivatives
 - =3 Do not solve problem after checking only first derivatives
 - =4 Solve problem after checking first and second derivatives
 - =5 Do not solve problem after checking first and second derivatives
- If the user wishes to check his first or second partial derivatives by having them printed along with experimental SUMT's numerical approximations, he can accomplish this by appropriately setting NEXOP1.
- In all cases the first values printed out are those gotten from the user's code, and the second values are those gotten by numerical differencing.
- NEXOP2 Key for choosing unconstrained minimization technique as follows:
- =1 The method for minimizing the unconstrained penalty function is to be the generalized Newton-Raphson method as modified to handle indefinite Hessian matrices. This method requires function values, first and second derivatives.
 - =2 Same as 1, except that when an "orthogonal move" is made because of an indefinite Hessian matrix $-VP$ is added to the orthogonal move vector.
 - =3 Steepest descent is used to minimize P-function.
 - =4 The method for minimizing the unconstrained penalty function is McCormick's modification of the Fletcher-Powell method as reported in the Fiacco-McCormick book (1). This requires function values and first derivatives.

b. Default Values of NAMELIST Data Items

Variables in \$DATA are given the following values in SUMT prior to execution of the NAMELIST read.

```
EPSI      : 1.E-9
RHOIN     : 1.
THETA0    : 1.E-6
RATIO     : 16.
TMMAX     : 30.
M         : 0
N         : 0
MZ        : 0
X(I)      : 0.  I=1,...,100
NT(1)     : 3
NT(2)     : 2
NT(3)     : 1
NT(4)     : 1
NT(5)     : 3
NT(6)     : 1
NT(7)     : 1
NT(8)     : 1
NT(9)     : 1
NT(10)    : 1
KEP1      : 1.E-4
KEP2      : 0.
NEXOP1    : 1
NEXOP2    : 4
```

D. Program Description

1) Usage:

The program consists of a main routine, two control subroutines (BODY, FEAS), twenty one special purpose subroutines (CONVRG, ESTIM, EVALU, FINAL, GRAD, INVERS, OPT, OUTPUT, PEVALU, SAVE, REJECT, RHOCOM, SECORD, STORE, TCHECK, TIMEC, SET, XMOVE, DIFF1, DIFF2, CHCKER) and three user supplied subroutines (RESTNT, GRAD1, MATRIX). Input is coordinated by the main routine.

Output is from the main routine and subroutines BODY, CHCKER, CONVRG, ESTIM, FEAS, INVERS, OPT, OUTPUT, PUNCH, TCHECK, TIMEC.

2) Subroutines Required:

SUBROUTINE BODY coordinates all subroutines.

SUBROUTINE CHCKER computes first derivatives of objective function using GRAD1 and DIFF1.

SUBROUTINE CONVRG(N1) checks for convergence.

SUBROUTINE DIFF1(IN) computes first derivatives by central difference.

SUBROUTINE DIFF2(IN) computes second derivatives by central difference.

SUBROUTINE ESTIM estimates Lagrange multiplier (λ_i) values and final solution extrapolation.

SUBROUTINE EVALU evaluates objective function and constraints.

SUBROUTINE FEAS determines feasibility of starting point; if not feasible, a feasible point is sought; if no feasible point possible, error message printed.

SUBROUTINE FINAL(N2) checks for convergence.

SUBROUTINE GRAD(IS) computes first derivatives of penalty function.

SUBROUTINE INVERS(NSME) solves linear system of equations.

SUBROUTINE OPT performs one dimensional search by Golden Section method (modified Fibonacci).

SUBROUTINE OUTPUT (K) prints out results of each iteration.

SUBROUTINE PEVALU computes value of penalty function and dual.

SUBROUTINE REJECT returns stored values to original locations.

SUBROUTINE RHOCOM computes initial value of r.

SUBROUTINE SAVE writes a NAMELIST data description of the end conditions to device NS. In event of execution abort due to excessive computation time (TMMAX)

SUBROUTINE SECORD(IS) computes second derivatives of penalty function.

SUBROUTINE SET(TMAX) stores the time at start of run.

SUBROUTINE STORE stores values of current point.

SUBROUTINE TCHECK checks run time for 90% of maximum specified.

SUBROUTINE TIMEC checks elapsed time of run for possible termination.

SUBROUTINE XMOVE determines search directions by one of several options (modified Newton-Raphson, Steepest Descent, Fletcher-Powell).

SUBROUTINE RESTNT(IN,VAL) specifies objective function and constraints (user supplied).

SUBROUTINE GRAD1(IN) evaluates analytical first derivatives for objective function and constraints (user supplied).

SUBROUTINE MATRIX(J,L) evaluates analytical second derivatives for objective function and constraints (user supplied).

3) DIMENSION Requirements:

The general subroutines in the code are currently dimensioned to handle a problem with up to 100 variables (N) and 200 constraints (M+MZ). This should be sufficient for most problems attempted with this method. If the limits need to be changed, a detailed description of the various parameters are given elsewhere (2).

4) Output

The program title is printed first followed by a listing of the parameters, option keys, and starting point information. This is followed by intermediate result printouts until convergence is achieved. At convergence, the final answer information is printed. Execution time is listed with the intermediate results. In the event that the execution time limit set by TMAX is attained, the stopping point information needed to recommence computation at the same point achieved is output to device NS in NAMELIST form. This file (if generated) may then be cataloged and used as the input NAMELIST data in a subsequent execution.

E. Example Problem

The following minimization problem illustrates the effective use of the SUMT library routines:

$$\text{Minimize} \quad F(X_1, X_2) = (X_1 - 1.)^2 + (X_2 - 2.)^2$$

Subject to

$$C_1 : X_1 + X_2 \leq 2.$$

$$C_2 : X_2 - X_1 = 1.$$

$$C_3 : X_1 \geq 0.$$

$$C_4 : X_2 \geq 0.$$

Here, the objective function is nonlinear, while all constraints are linear. Constraint C1 is an inequality, C2 is an equality, and C3 and C4 are trivial inequality constraints. Imposing the constraint format defined in Section A, constraint C1 is re-written as

$$C_1 : 2. - X_1 - X_2 \geq 0.$$

and constraint C2 becomes

$$C_2 : X_2 - X_1 - 1. = 0.$$

After selecting initial values for the X vector, and choosing to employ the generalized Newton-Raphson unconstrained optimization algorithm, the computer run is coded as (see next page)

11

```

QTS,T35,I0100,CML00000. T820163,FLOYD
ATTACH,SUMT,SUMT,ID=T800535.
LIBRARY,SUMT.
FTN.
LGO.
*SOR
  PROGRAM MAIN(INPUT,OUTPUT,SAVE,TAPE5=INPUT,TAPE6=OUTPUT,
1  TAPE7=SAVE)
  COMMON/DEVC/NI,NO,NS
  NI=5
  NO=6
  NS=7
  CALL SUMT
  STOP
  END
  SUBROUTINE RESTNT(IN,VAL)
  COMMON/SHARE/X(100),DEL(100),A(100,100),N(5)
  IF(IN) 10,10,20
10  VAL=(X(1)-1.)**2+(X(2)-2.)**2
  RETURN
20  GO TO(21,22),IN
21  VAL=2.-X(1)-X(2)
  RETURN
22  VAL=X(2)-X(1)-1
  RETURN
  END
*ECR
  $DATA N=2,M=1,MZ=1,X=0.,1.,NT(2)=1,NT(5)=2,NT(8)=2,NEXOP2=1 $END
*EOF

```

The solution was obtained in about 0.25 seconds of Central Processor time. The final printout is

```

FINAL VALUE OF F = 5.00000004E-1
FINAL X VALUES
X( 1) = 4.99999998E-1 X( 2) = 1.50000000E0
FINAL CONSTRAINT VALUES
G( 1) = 3.72349973E-9 G( 2) = 5.54578605E-11

```

REFERENCES

- (1) Fiacco, A.V., and G.P. McCormick. Nonlinear Sequential Unconstrained Minimization Techniques. John Wiley and Sons, Inc., New York, 1968.
- (2) Mylander, W.C., R.L. Holmes, and G.P. McCormick. A Guide to SUMT-Version 4: The Computer Program Implementing the Sequential Unconstrained Minimization Technique for Nonlinear Programming. Research Analysis Corporation, McLean, Virginia, 1971. (AD731 391)
- (3) Kuester, J.L., and J.H. Mize. Optimization Techniques with FORTRAN. McGraw-Hill Book Company, New York, 1973.
- (4) Fiacco, A.V., and G.P. McCormick. "Computational Algorithm for the Sequential Unconstrained Minimization Technique for Nonlinear Programming," Management Science (10), 601-617 (July 1964).
- (5) Bracken, J., and G.P. McCormick. Selected Applications of Nonlinear Programming. John Wiley and Sons, Inc., New York, 1968.

The following program, STCF CV, is a Fortran Four executive program for SUMT. This program is run to calculate the maximum CF for given contours of CV. In its current form, the program will request a data entry in the form of a "namelist" for each CV contour value. These contours are set at two unit increments and the user enters the starting value.

```

      PROGRAM STCF CV(INPUT,OUTPUT,NSMT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7
+ =NSMT)
      COMMON/DEVC/N1,N0,NS
      COMMON/CONTUR/CVI
C
C
C      THIS PROGRAM USES SUMT TO SOLVE FOR THE MAXIMA OF RESPONSE ONE,
C      CF, CONSTRAINED TO A VALUE,CVI, OF RESPONSE TWO. THE PROBLEM
C      IS FURTHER RESTRICTED BY THE OPERATING DOMAIN UNDER CONSIDERSTION.
C
C
C
C      N1=5
C      N0=6
C      NS=7
C      II=0
C
C      WRITE(N0,13)
13  FORMAT('*** ENTER INITIAL CVI VALUE***')
      READ(N1,*)CVI
C      CVI= 1122.0
      DO 19 II=1,34
C
C      CVI= CVI+ 2.0
C
C      WRITE(N0,11) CVI
      WRITE(NS,11) CVI
11  FORMAT(1X,F7.2,'=*CVI*FOR THE FOLLOWING RUN *****')
      CALL SUMT
      WRITE(NS,12) CVI
12  FORMAT(1X,F7.2,'%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%')
      WRITE(N0,12) CVI
19  CONTINUE
C
C      STOP
      END

```



```

C
SUBROUTINE RECTNT(IN,VAL)
COMMON/SHARE/X(100),DEL(100),A(100,100),N(5)
COMMON/CONTUR/CUI
C
C
IF(IN) 10,10,20
10 VAL=-504.542366-0.4715619*X(1)-0.2955107*X(2)-0.0288939*X(3)
C   -0.0278609*X(4)-0.0000003*(X(1)**2)+0.0000671*(X(2)**2)
C   +0.0000006*(X(3)**2)+0.0000005*(X(4)**2)+0.0000015*X(1)*X(3)
C   +0.0000009*X(1)*X(4)+0.0000007*X(2)*X(3)+0.0000006*X(2)*X(4)
C   +0.0000013*X(3)*X(4)
RETURN
C
20 GO TO(22,23,24,25,26,27,28,29,30),IN
C
22 VAL= 450 - X(1)
RETURN
23 VAL= X(1)-300
RETURN
24 VAL= 750 - X(2)
RETURN
25 VAL= X(2)-450
RETURN
26 VAL= 1040 - X(3)
RETURN
27 VAL= X(3)-720
RETURN
28 VAL= 424 - X(4)
RETURN
29 VAL= X(4)-200
RETURN
30 VAL=(1027.039-CUI)+0.1658222*X(1)+0.1198222*X(2)+0.0307574*X(3)
C   +0.0019064*X(4)+0.0000054*(X(4)**2)-0.0000153*(X(1)**2)
C   -0.0000040*(X(2)**2)-0.0000075*(X(3)**2)-0.0000129*X(1)*X(2)
C   -0.0000175*X(1)*X(3)-0.0000087*X(2)*X(3)-0.0000054*X(3)*X(4)
RETURN
END
/

```

An example of the SUMT output for the final iteration to maximize CF for a CV value of 1160 is shown on the next page.

SOLUTION OF THE SUBPROBLEM

F = -0.1737214E+03 P = -0.1737214E+03 G = -0.17372135E+03

VALUES OF X VECTOR

X(1) = 0.441174E+03 X(2) = 0.4500610E+03 X(3) = 0.10034332E+04
X(4) = 0.541200E+03 X(5) = 0.541200E+03 X(6) = 0.541200E+03

VALUES OF THE CONSTRAINTS

G(1) = 0.121360E-02 G(2) = 0.1017260E-06 G(3) = 0.5017314E-07
G(4) = 0.210110E-03 G(5) = 0.417200E-06 G(6) = 0.5313557E-07
G(7) = 0.511112E-06 G(8) = 0.70575542E-07 G(9) = 0.40515746E+01

FINAL VALUE OF F = -0.1737214E+03

FINAL X VALUES

X(1) = 0.441174E+03 X(2) = 0.4500610E+03 X(3) = 0.10034332E+04
X(4) = 0.541200E+03 X(5) = 0.541200E+03 X(6) = 0.541200E+03

FINAL CONSTRAINT VALUES

G(1) = 0.121360E-02 G(2) = 0.1017260E-06 G(3) = 0.5017314E-07
G(4) = 0.210110E-03 G(5) = 0.417200E-06 G(6) = 0.5313557E-07
G(7) = 0.511112E-06 G(8) = 0.70575542E-07 G(9) = 0.40515746E+01
11 0.000000E+00

NOTE: Since the SUMT is a minimizing algorithm, the minimum of the negative CF was calculated. Therefore, the final CF objective function, f, is negative for this maximization.

APPENDIX E

Analysis Data

This appendix first contains the program "surface" which generated data for the CF and CV plots. Following the program are the enlarged contour plots for each MOE and the enlarged overlay plots used in Chapter Four. Likewise, the enlarged MOE trade-off graphs are included.

Finally, Tables IX through XII contain the actual SUMT data for each of the MOE trade-offs in Chapter Four and also the AF trade-off cases.

```

PROGRAM SURFACE
C
C
OPEN(11,FILE='PLOT')
REWIND 11
IW2=450
WRITE(11,01)
01  FORMAT(3X,'DEF',3X,' W1',3X,' W2 ',3X,' W3',3X,' W4')
C
C  LOOP CF,W1 AND,W2 THROUGH THEIR RESPECTIVE RANGES TO SOLVE FOR W
C
DO 23,ICF=800,920,20
DO 22,IW1=300,450,25
DO 21,IW4=200,420,20
W3=(1/0.0289)*(ICF-504.5424-0.4716*IW1-0.2995*IW2-0.02791*IW
C  +0.0001*IW2**2)
IF(W3.LT.1041.0 .AND. W3.GE.720.0) THEN
WRITE(11,02) ICF,IW1,IW2,W3,IW4
02  FORMAT(2X,I3,2X,I3,2X,I3,2X,F6.1,2X,I3)
ENDIF
21  CONTINUE
22  CONTINUE
23  CONTINUE
C
WRITE(11,03)
03  FORMAT(5X,'*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*')
WRITE(11,04)
04  FORMAT(3X,' CV ',2X,' W1 ',2X,' W2 ',2X,' W3 ',2X,' W4')
C
C  LOOP CV,W3,AND W2 THROUGH THEIR RESPECTIVE RANGES TO SOLVE FOR W
C
DO 33,ICV=1155,1225,10
DO 32,IW1=300,450,25
DO 31,IW4=200,420,20
W3=(1/0.0303)*(ICV-1027.039-0.1658*IW1-0.1198*IW2-0.0019*IW4
IF(W3.LT.1041.0 .AND. W3.GE.720.0) THEN
WRITE(11,05) ICV,IW1,IW2,W3,IW4
05  FORMAT(2X,I4,2X,I3,2X,I3,2X,F6.1,2X,I3)
ENDIF
31  CONTINUE
32  CONTINUE
33  CONTINUE
C
PRINT *, 'RUN COMPLETE-CONTOUR MAP DATA IN FILE-PLOT.'
CLOSE(11)
END

```

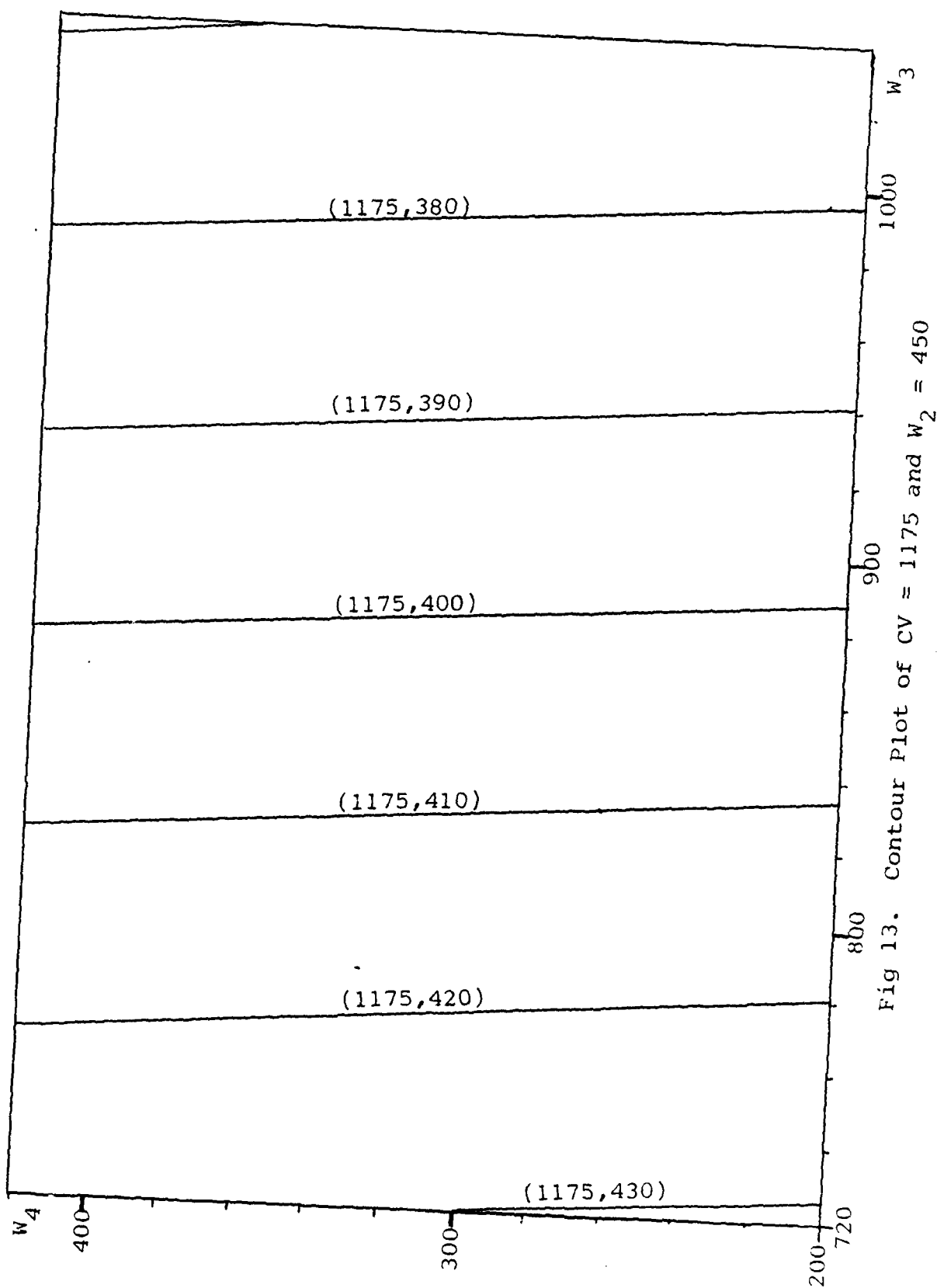
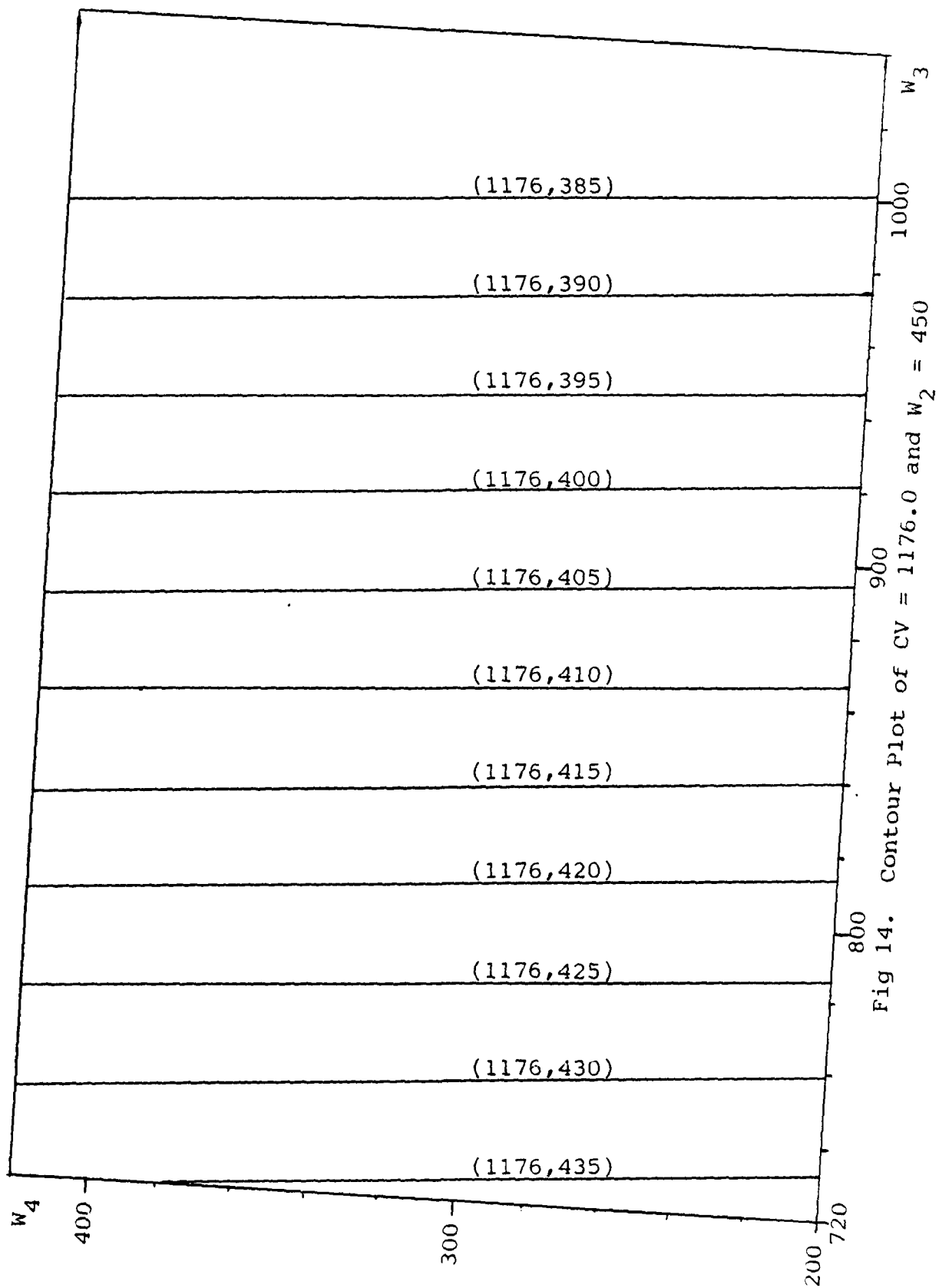


Fig 13. Contour Plot of $CV \approx 1175$ and $W_2 = 450$



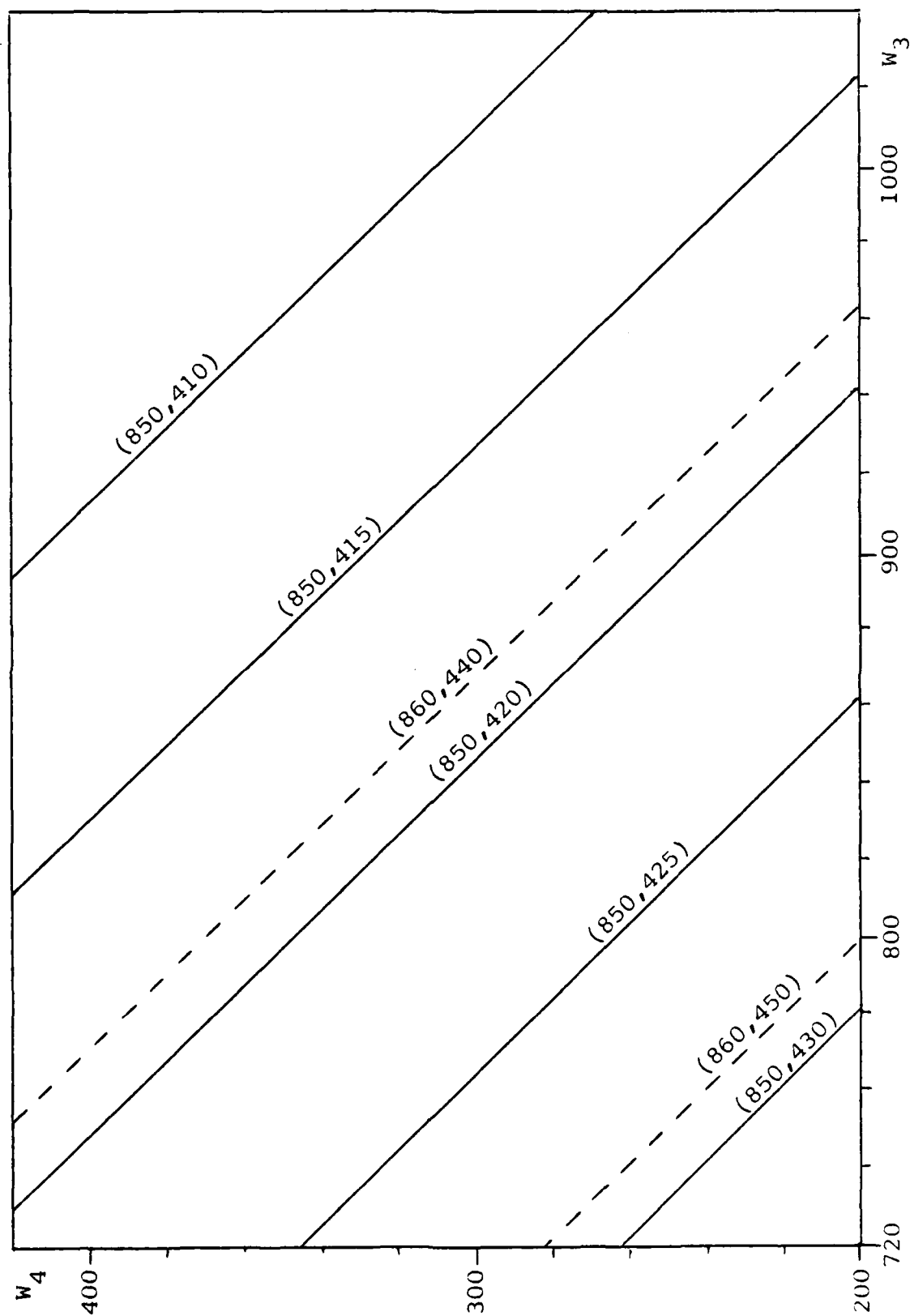


Fig 15. Contour Plot of $CF = 850/860$ and $W_2 = 450$

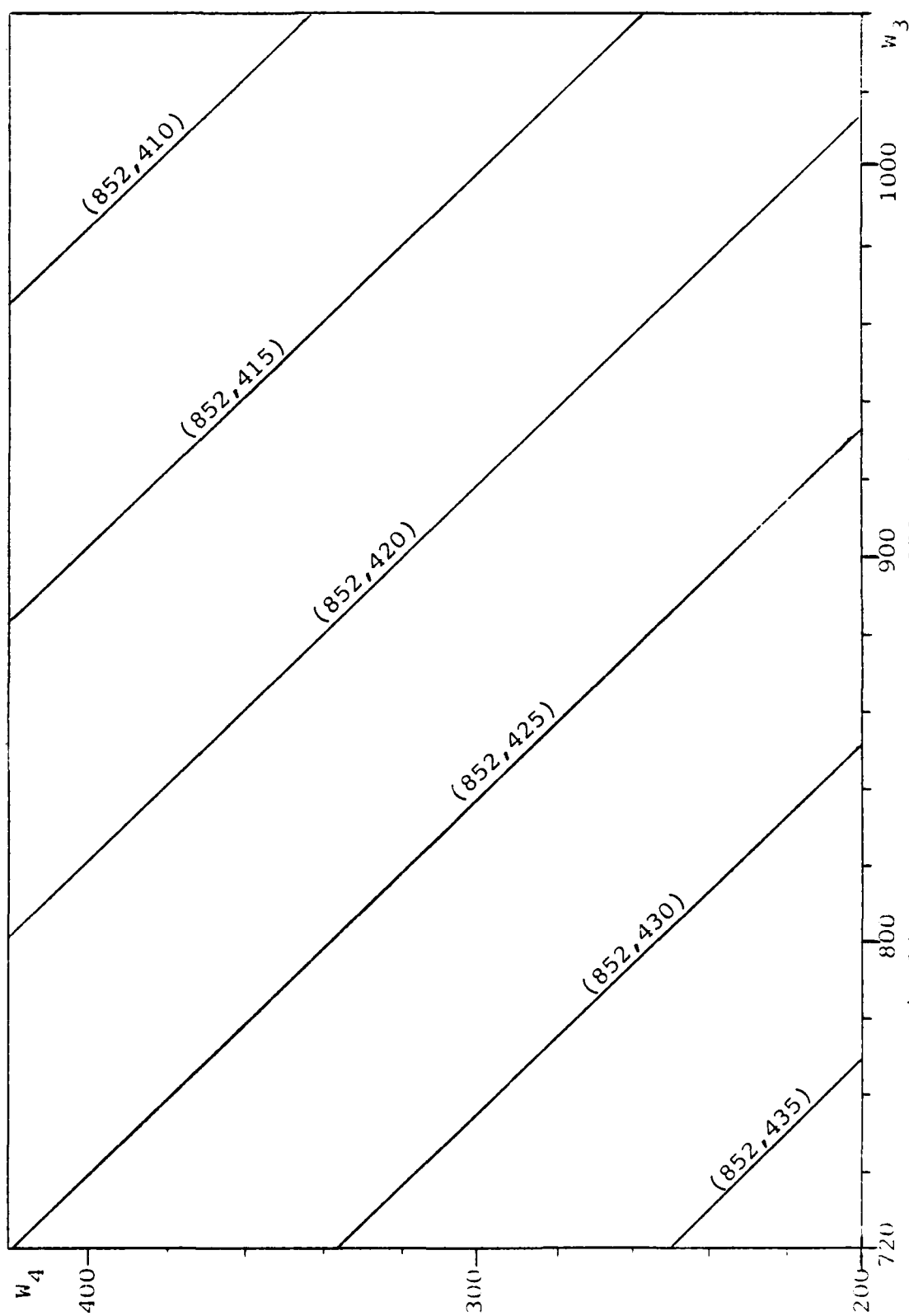


Fig 16. Contour Plot of $CF = 852$ and $W_2 = 450$

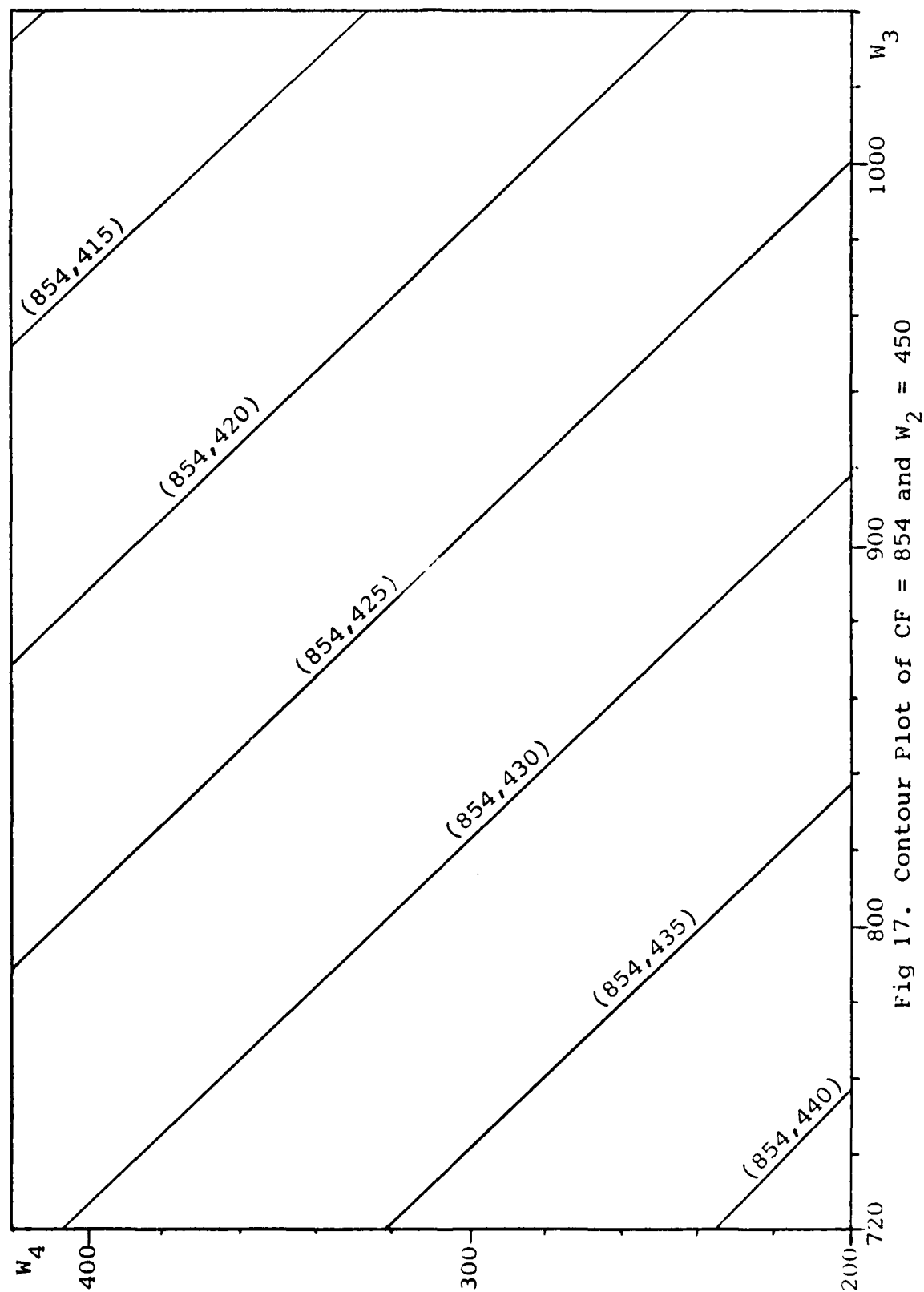


Fig 17. Contour Plot of $CF = 854$ and $w_2 = 450$

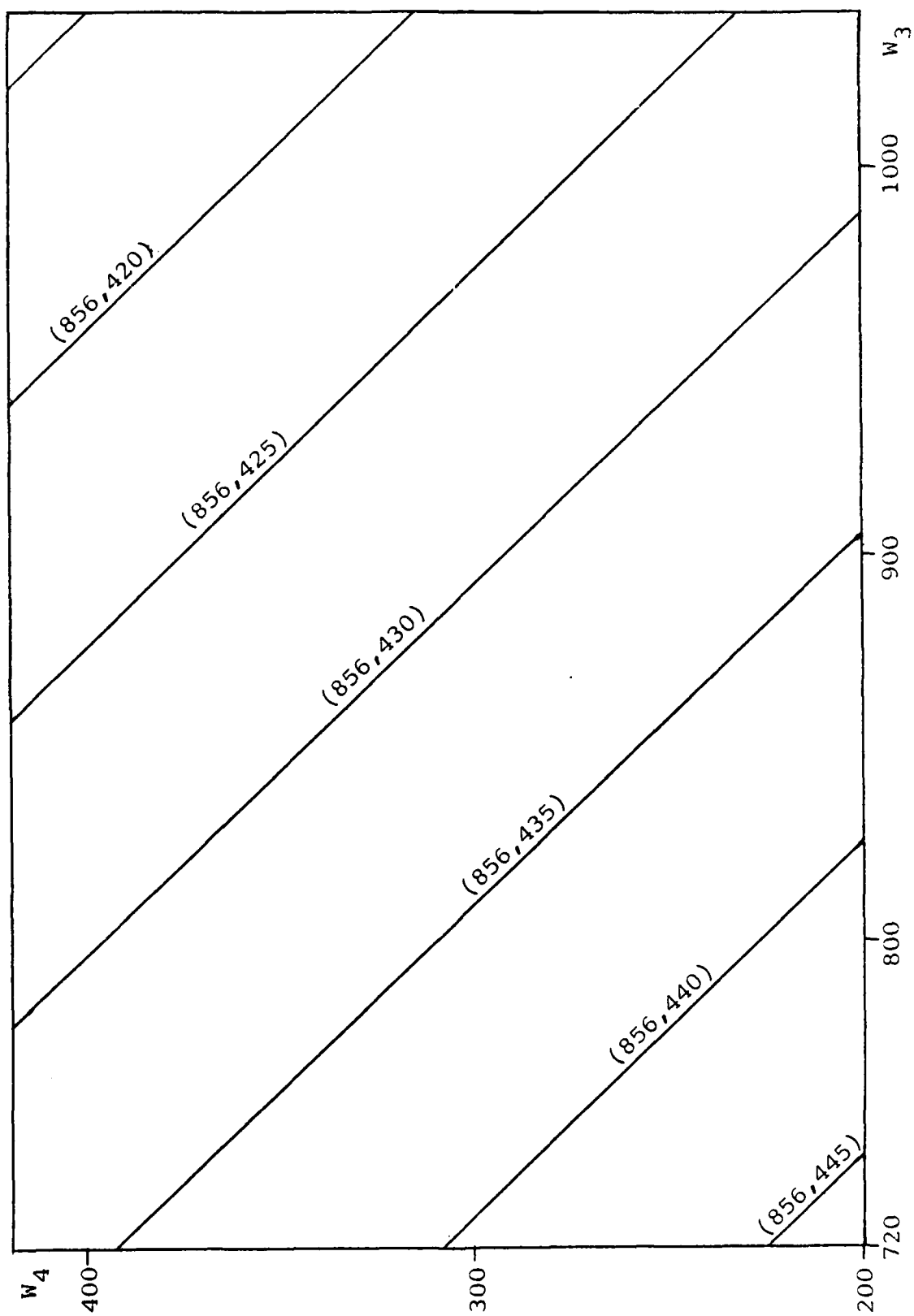
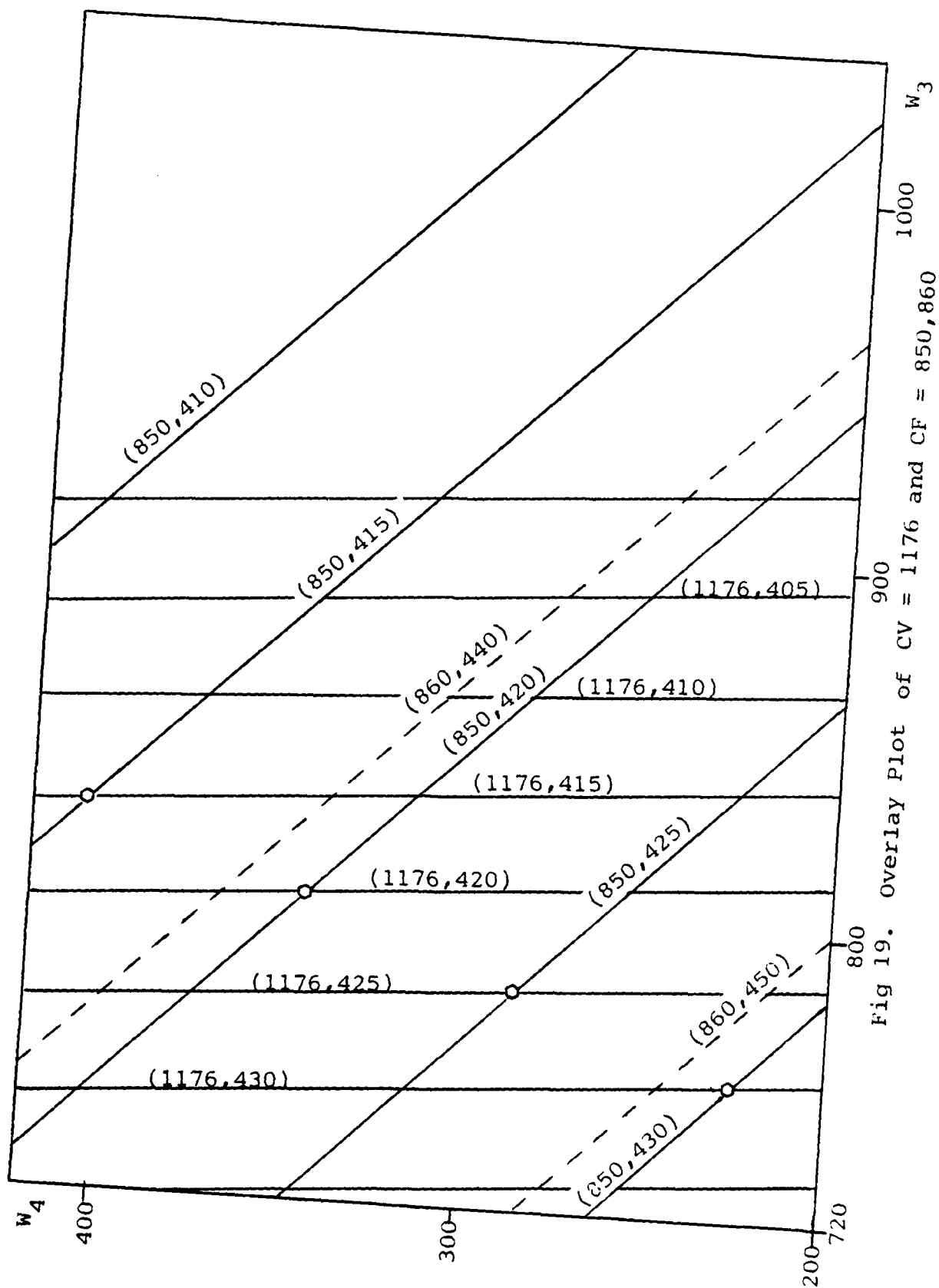


Fig 18. Contour Plot of $CF = 856$ and $W_2 = 450$



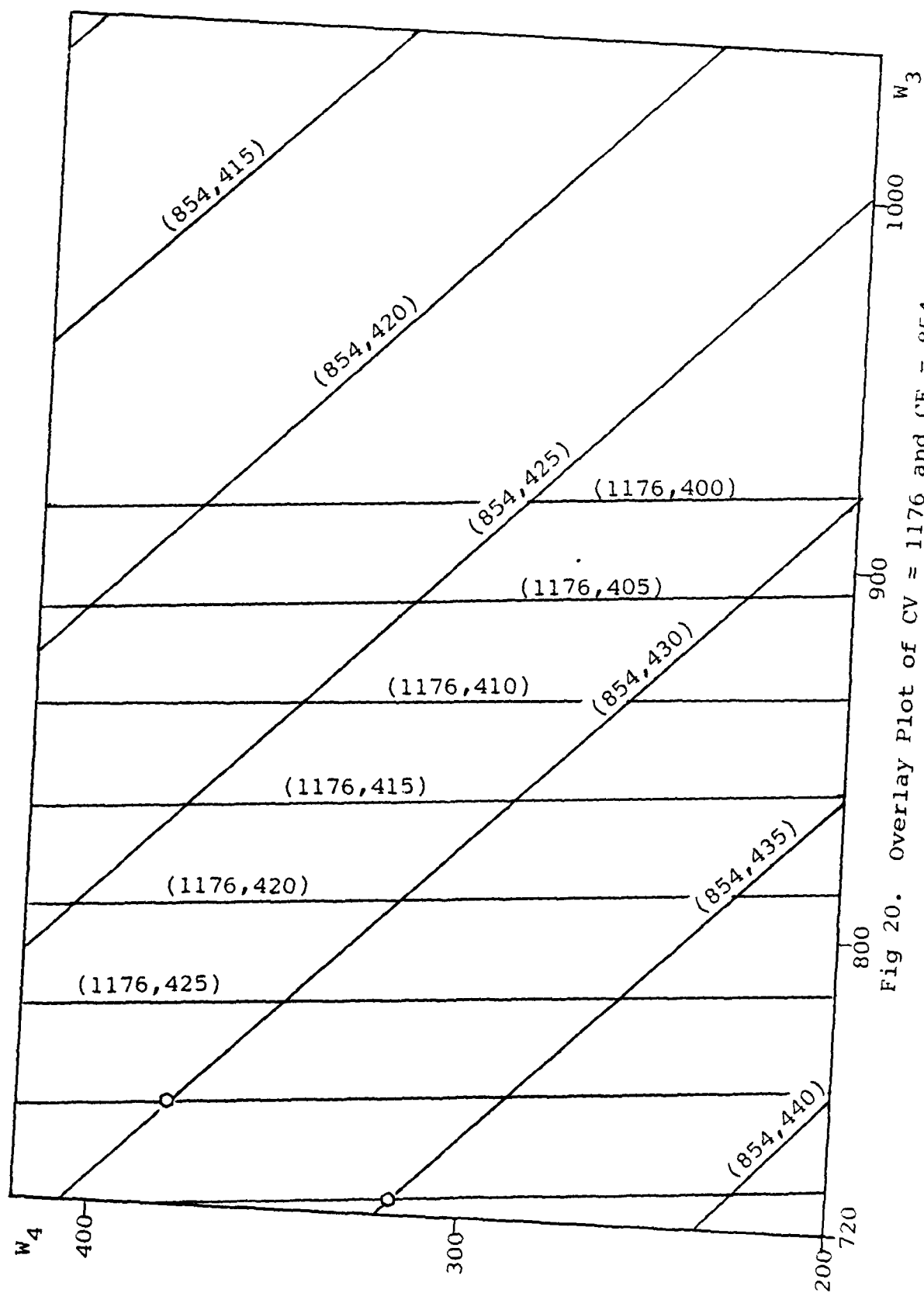


Fig 20. Overlay Plot of $CV = 1176$ and $CF = 854$

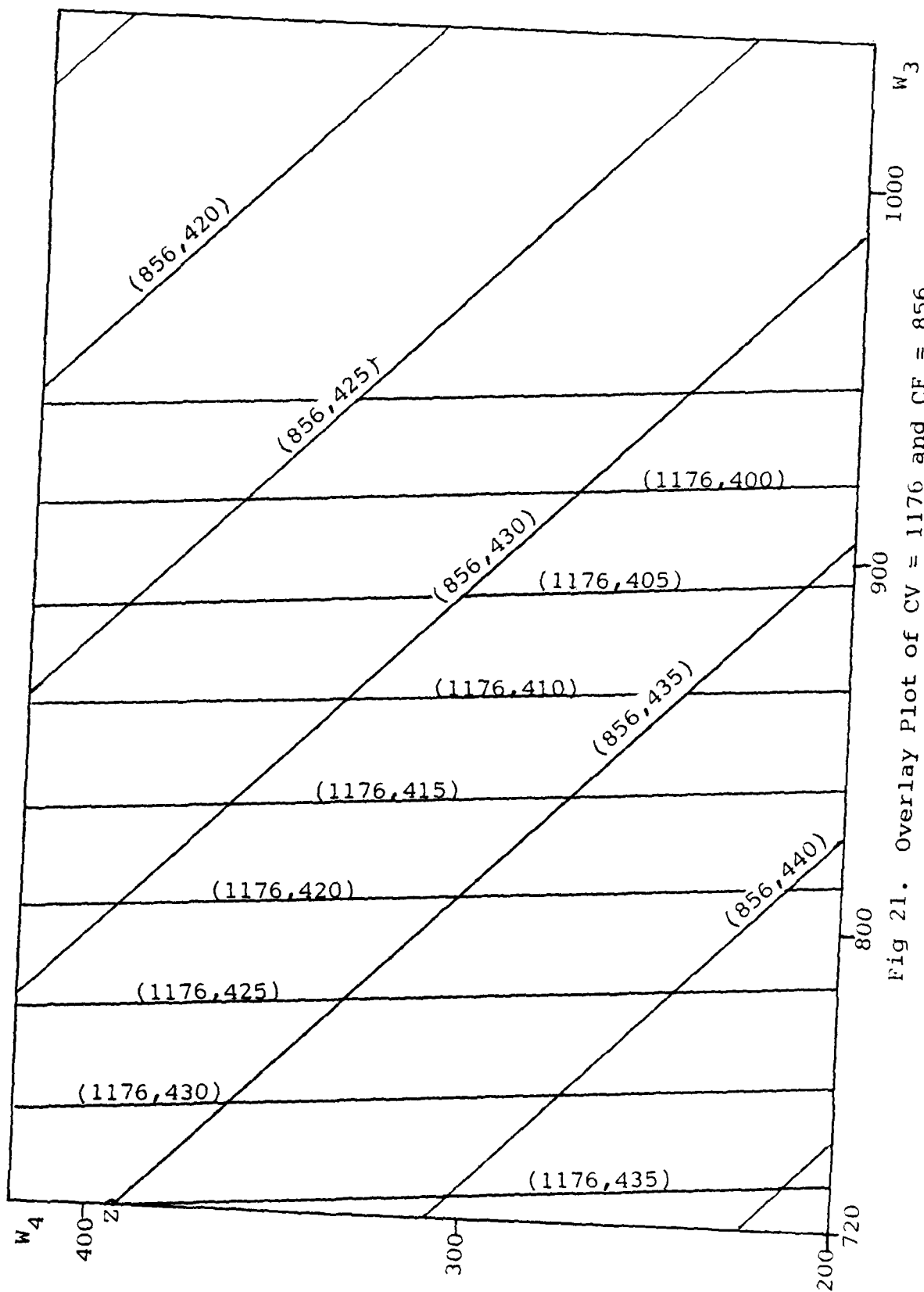


Fig 21. Overlay Plot of $CV = 1176$ and $CF = 856$

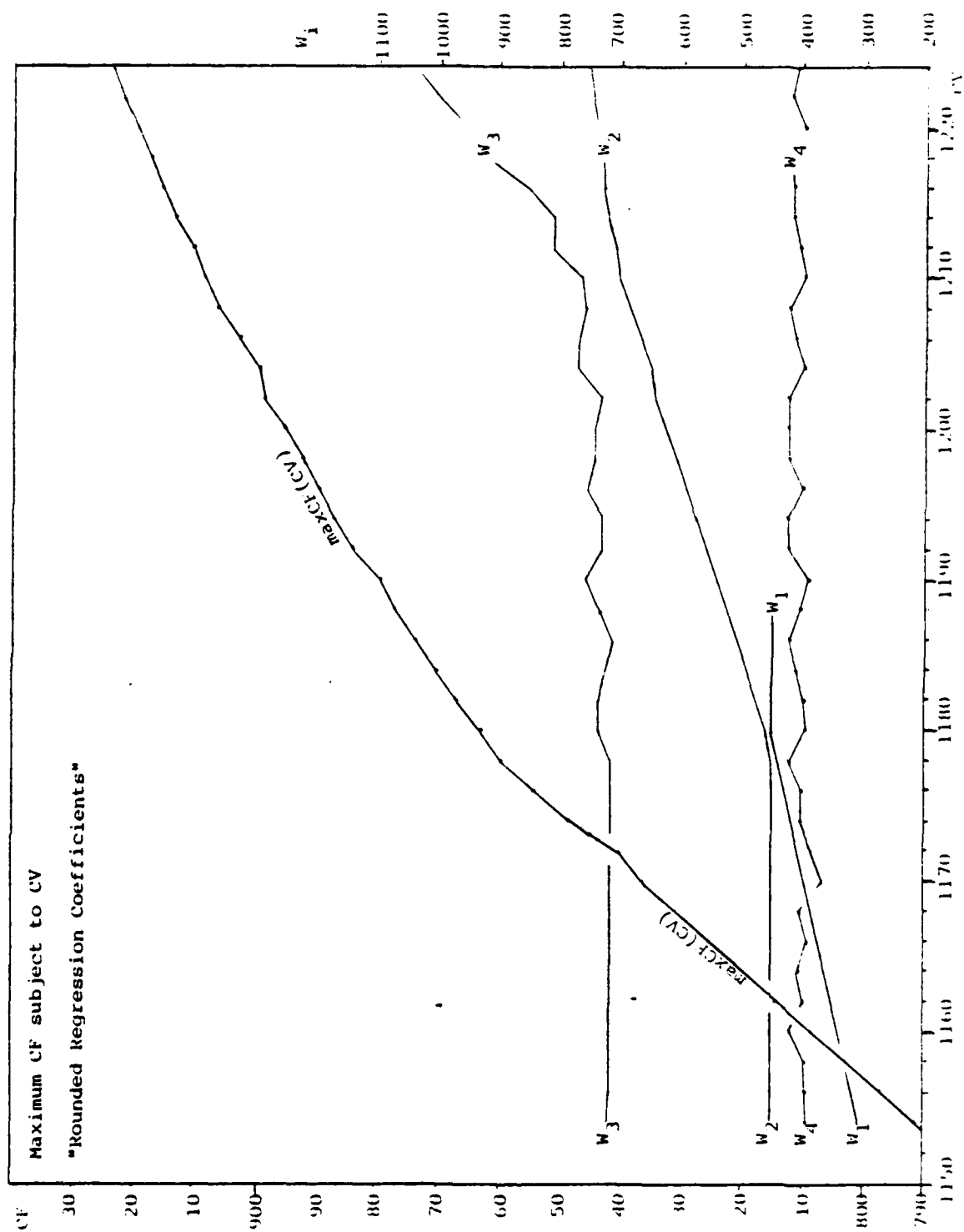


Fig 22. Maximum CF subject to CV

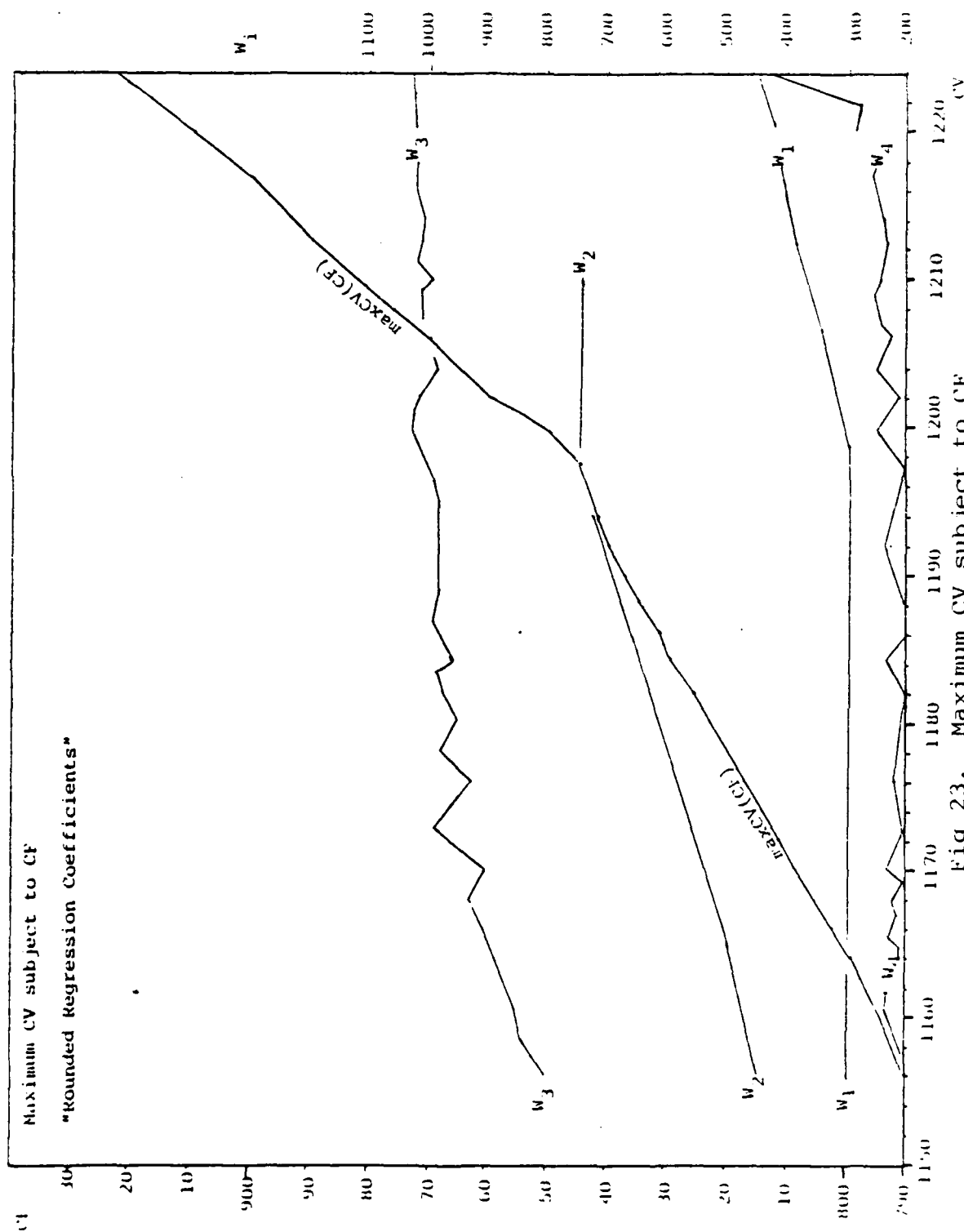


Fig 23. Maximum CV subject to CF

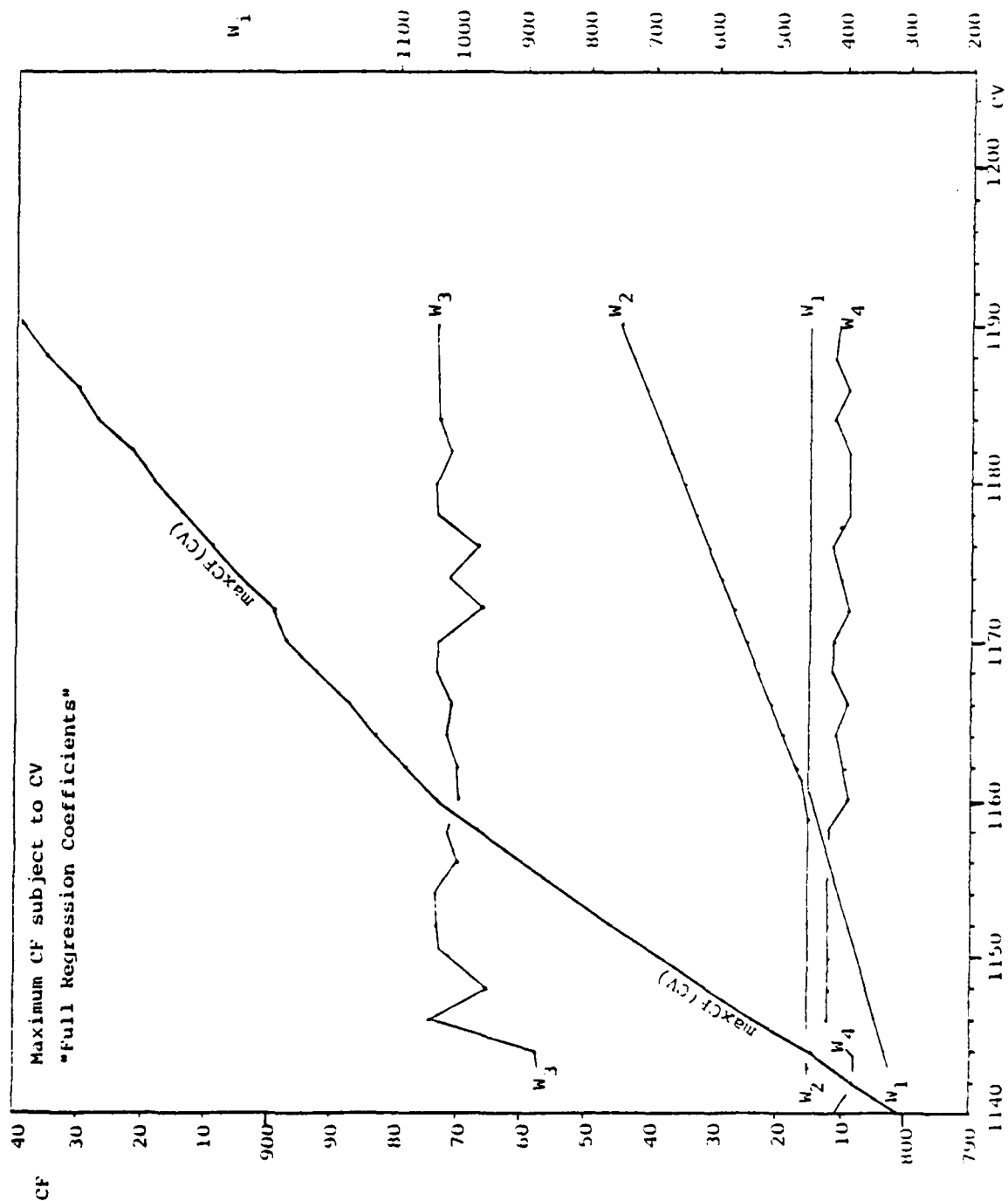


Fig 24. Maximum CF subject to CV Full Regression Coefficients

TABLE IX

SUMT: Maximum CF given CV

Max CF	CV	W_1	W_2	W_3	W_4
791.14	1154	300.90	450.00	727.41	390.96
797.32	1156	314.39	450.00	720.06	393.68
803.05	1158	326.44	450.00	720.04	395.05
809.27	1160	338.04	450.00	724.90	420.98
814.42	1162	350.43	450.00	720.65	396.45
820.30	1164	362.52	450.00	720.06	403.47
825.45	1166	374.46	450.00	721.93	384.54
831.71	1168	386.64	450.00	720.03	404.99
836.54	1170	399.13	450.00	720.04	366.74
840.10	1172	412.27	450.01	720.03	272.27
848.70	1174	422.86	450.00	720.04	401.55
855.36	1176	434.94	450.00	720.05	400.34
860.47	1178	446.60	450.00	720.09	421.44
863.35	1180	450.00	457.23	741.20	392.99
866.67	1182	450.00	473.03	744.62	393.85
870.81	1184	450.00	493.35	729.32	413.37
874.32	1186	450.00	511.60	722.72	421.92
876.92	1188	450.00	523.33	742.77	409.83
879.39	1190	450.00	536.63	757.00	393.54
883.63	1192	450.00	558.53	734.97	422.14
886.68	1194	450.00	575.42	734.21	421.97
888.70	1196	450.00	587.05	755.41	397.80
892.42	1198	450.00	605.34	747.59	423.83
895.31	1200	450.00	622.92	744.21	422.81
898.31	1202	450.00	642.42	733.31	422.44
899.78	1204	450.00	649.79	771.48	391.81
903.12	1206	450.00	656.37	770.46	415.01
906.12	1208	450.00	686.26	757.65	421.90

continued

Table IX continued

907.89	1210	450.00	701.26	766.06	392.20
910.30	1212	450.00	705.96	811.95	405.11
913.11	1214	450.00	722.22	812.98	415.63
915.14	1216	450.00	729.03	851.64	411.73
917.24	1218	450.00	728.49	918.16	421.16
918.70	1220	450.00	736.36	954.09	394.16
921.40	1222	450.00	741.16	998.81	419.85
923.19	1224	450.00	748.56	1035.78	406.18

TABLE X
SUMT: Maximum CV given CF

Max CV	CF	W_1	W_2	W_3	W_4
1156.85	790	300.69	456.56	806.43	225.08
1158.72	792	300.10	462.81	846.26	219.77
1160.91	794	300.03	467.21	901.65	203.39
1160.65	796	300.05	478.98	844.41	249.25
1163.77	798	300.03	480.44	942.41	209.29
1165.03	800	300.06	492.10	938.52	201.72
1165.48	802	300.06	502.08	912.54	230.11
1166.98	804	300.44	510.39	927.30	222.20
1168.28	806	300.38	518.72	937.08	227.26
1169.72	808	300.07	536.28	918.73	203.67
1170.33	810	300.24	542.09	912.69	239.71
1173.13	812	300.38	545.69	990.97	203.77
1174.13	814	300.03	561.75	963.09	203.99
1175.35	816	300.04	573.60	956.37	205.12
1176.17	818	300.01	586.11	933.72	220.24
1178.12	820	300.02	589.45	983.99	218.42
1179.31	822	300.39	601.34	974.51	218.53
1180.71	824	300.02	616.73	962.25	213.37
1182.49	826	300.02	627.16	980.34	202.34
1183.86	828	300.29	636.29	987.55	206.58
1184.49	830	300.02	648.30	960.61	238.85
1186.93	832	300.02	659.63	997.97	204.95
1188.22	834	300.04	675.27	978.87	205.20
1189.88	836	300.01	685.26	994.07	204.56
1191.43	838	300.00	697.02	998.58	205.44

continued

Table X continued

1192.24	840	300.01	706.00	988.06	237.85
1193.80	842	300.24	718.39	989.37	236.49
1195.41	844	300.32	731.12	991.56	235.77
1197.93	846	300.02	747.69	1012.72	203.01
1199.14	848	300.58	749.83	1039.14	226.71
1199.01	850	305.98	748.98	1007.94	243.83
1199.33	852	310.22	749.25	993.86	256.96
1201.03	854	312.26	749.99	1035.67	247.04
1201.72	856	317.12	749.15	1035.57	241.01
1202.64	858	322.60	749.95	1034.44	217.00
1202.74	860	325.91	749.14	1020.92	251.00
1203.20	862	330.81	749.92	1006.25	251.01
1203.58	864	335.20	749.95	994.36	260.70
1204.23	866	339.66	749.95	991.37	260.00
1205.05	868	343.32	749.29	1000.67	263.64
1206.74	870	347.41	749.19	1036.13	230.08
1207.41	872	350.57	749.83	1037.53	243.50
1208.07	874	354.37	749.94	1037.51	250.42
1208.78	876	358.96	749.95	1036.07	245.94
1208.60	878	364.25	749.94	1000.83	264.71
1209.29	880	368.66	749.95	999.48	263.19
1210.87	882	371.46	749.57	1038.10	249.54
1210.73	884	376.69	749.95	1002.66	267.59
1211.67	886	380.79	749.93	1011.75	260.63
1211.99	888	384.72	749.37	1002.17	278.75
1212.56	890	392.34	747.70	988.09	244.99
1213.94	892	393.18	749.95	1018.63	259.10
1214.94	894	395.98	749.68	1036.51	266.23

continued

Table X continued

1215.65	896	400.98	749.94	1032.46	256.32
1216.46	898	404.98	749.95	1037.14	255.45
1217.10	900	408.89	749.95	1036.68	261.57
1217.32	902	414.12	749.25	1017.83	267.99
1217.47	904	419.60	749.28	993.09	272.62
1219.34	906	422.18	749.95	1038.37	250.20
1219.84	908	425.93	749.16	1036.78	264.29
1220.04	910	428.75	749.05	1026.34	299.74
1220.63	912	433.66	749.95	1015.91	294.55
1221.94	914	437.33	749.93	1039.70	279.57
1222.20	916	439.04	749.90	1036.08	326.19
1222.20	918	442.96	749.07	1016.19	356.60
1223.39	920	445.30	749.72	1039.40	361.37
1223.57	922	448.49	749.94	1025.31	392.38
1224.33	924	449.99	749.98	1039.90	423.55

TABLE XI

SUMT: Maximum CF given AF

Max CF	AF	W_1	W_2	W_3	W_4
796.13	1440	321.83	452.41	727.13	200.04
799.73	1445	330.88	450.04	720.76	200.04
802.53	1450	336.80	450.05	721.00	200.02
804.89	1455	341.44	450.03	727.01	200.03
808.22	1460	348.89	450.11	720.45	200.03
810.83	1465	354.23	450.07	723.60	200.03
813.63	1470	360.08	450.22	723.75	200.21
816.69	1475	366.78	450.08	721.63	200.03
819.61	1480	372.93	450.28	720.74	200.03
822.43	1485	378.83	450.39	721.03	200.23
824.85	1490	383.49	450.69	726.90	200.03
827.98	1495	390.44	450.32	724.42	200.00
831.09	1500	397.13	450.56	720.81	200.37
834.03	1505	403.47	450.28	721.48	200.00
836.54	1510	408.15	451.23	724.47	200.64
838.33	1515	409.29	456.35	730.19	202.12
842.83	1520	422.09	450.40	721.21	200.00
844.41	1525	421.73	457.78	729.56	200.14
847.60	1530	429.29	456.07	728.20	200.47
851.31	1535	439.78	450.37	726.31	200.01
854.77	1540	447.52	450.32	720.09	200.03
857.14	1545	449.97	456.00	721.49	200.08
858.87	1550	449.73	464.28	726.03	201.45
860.38	1555	449.98	469.43	735.01	204.39
862.92	1560	449.98	482.99	733.27	200.12
864.93	1565	449.98	492.74	735.68	200.31
867.07	1570	449.98	503.75	735.30	200.27

continued

Table XI continued

868.61	1575	449.95	509.38	751.41	200.02
871.20	1580	450.00	524.54	739.16	200.34
873.10	1585	449.98	533.53	746.20	200.36
874.19	1590	449.98	534.88	772.94	202.43
876.57	1595	449.98	547.93	772.99	200.31
878.47	1600	449.99	556.52	783.66	200.22
880.76	1605	449.97	569.31	782.06	200.24
883.62	1610	449.98	588.61	760.89	200.19
884.32	1615	449.98	584.25	809.42	202.98
885.76	1620	449.98	588.42	829.18	207.62
888.59	1625	449.98	605.85	825.34	203.04
891.68	1630	449.89	628.21	803.24	200.03
886.08	1635	449.99	574.87	852.24	282.03
895.77	1640	449.99	646.63	835.78	200.57
898.28	1645	449.96	662.83	830.71	200.70
900.67	1650	449.98	676.19	838.94	200.09
903.28	1655	449.99	694.81	826.61	200.59
904.98	1660	449.99	702.35	836.39	209.36
905.97	1665	449.98	703.39	847.57	227.58
910.13	1670	449.98	731.02	866.00	206.81
910.13	1675	449.98	731.02	866.00	206.81
913.53	1680	449.97	741.14	908.44	231.17
914.25	1685	450.00	736.25	932.99	257.05
912.59	1690	449.99	730.61	854.22	309.40
918.45	1695	449.99	745.65	1003.14	285.49
917.32	1700	449.98	749.40	908.52	323.19
920.36	1705	449.97	747.56	1010.79	336.56
921.27	1710	449.99	749.51	1007.47	361.80
919.99	1715	449.98	745.02	949.79	399.31
920.62	1720	450.00	749.79	923.95	423.48

TABLE XII

SUMT: Maximum CV given AF

Max CV	AF	W ₁	W ₂	W ₃	W ₄
1157.03	1440	305.07	458.14	784.02	201.15
1157.90	1445	309.22	465.75	759.82	206.86
1158.70	1450	310.47	470.11	761.75	211.15
1159.54	1455	310.33	468.82	794.98	211.38
1160.71	1460	314.70	470.24	804.15	207.80
1162.22	1465	317.56	483.48	786.11	201.47
1163.16	1470	317.63	483.01	818.51	201.20
1163.59	1475	318.86	485.61	814.87	213.69
1165.36	1480	323.39	499.68	794.19	202.81
1166.34	1485	325.21	497.28	825.53	202.18
1167.59	1490	324.46	509.41	822.75	201.10
1168.66	1495	327.98	512.84	825.46	201.35
1169.66	1500	336.48	508.32	829.77	201.29
1170.85	1505	336.82	516.96	832.78	201.06
1171.97	1510	339.98	521.66	833.94	201.27
1173.39	1515	331.14	545.48	834.94	201.13
1174.09	1520	347.00	528.89	836.61	204.07
1175.85	1525	331.89	563.27	841.72	201.11
1176.55	1530	354.52	538.08	840.45	201.47
1177.26	1535	346.41	553.92	844.65	214.32
1179.63	1540	335.71	588.97	843.71	201.28
1179.22	1545	364.23	545.91	843.35	216.70
1182.43	1550	330.94	614.37	861.49	201.38
1183.72	1555	334.84	618.73	865.49	201.00
1185.29	1560	329.17	636.46	877.90	201.44
1185.34	1565	341.27	623.89	862.55	217.12

continued

Table XII continued

1186.87	1570	347.46	625.62	872.48	212.30
1188.43	1575	364.29	619.93	855.41	201.20
1190.64	1580	344.51	660.93	874.08	201.29
1190.45	1585	387.94	602.55	861.10	201.48
1191.17	1590	396.31	602.60	856.67	200.08
1195.90	1595	336.58	706.61	910.03	200.52
1197.38	1600	342.57	706.17	927.26	200.93
1199.17	1605	341.86	719.82	936.12	201.36
1201.69	1610	339.21	730.43	991.08	200.92
1201.03	1615	369.62	704.46	906.78	201.14
1204.96	1620	347.77	745.36	993.11	201.11
1205.27	1625	361.87	745.66	926.02	200.80
1206.77	1630	373.00	731.89	968.28	202.38
1207.57	1635	384.88	735.04	918.18	200.99
1209.94	1640	390.98	723.08	1008.67	202.00
1212.01	1645	390.98	743.71	995.71	200.74
1213.84	1650	396.73	749.46	1001.79	200.09
1215.48	1655	405.39	747.86	1015.02	200.05
1216.79	1660	413.96	749.80	1003.54	200.24
1218.21	1665	422.19	749.66	1005.76	201.92
1220.05	1670	430.96	749.81	1017.72	200.07
1219.98	1675	438.21	749.96	975.28	212.82
1221.39	1680	449.53	749.63	961.53	209.07
1222.87	1685	445.90	749.41	1023.35	235.69
1220.88	1690	447.89	749.93	948.86	268.78
1222.85	1695	447.86	749.95	1011.95	285.78
1223.06	1700	446.64	749.92	1023.81	315.50
1219.17	1705	449.31	749.92	880.35	355.75
1222.55	1710	446.30	749.94	1005.20	372.56
1224.09	1715	449.86	749.96	1035.14	387.36
1223.22	1720	449.18	749.94	1008.83	417.94

Bibliography

1. Binninger, G. C. and others. Mathematical Background and Programming Aids for the Physical Vulnerability System for Nuclear Weapons. Defense Documentation Center, DI-550-27-74, AD B010375L (November 1974).
2. Box, G. E. P. and Behnken, D. W. "Some New Three Level Designs for the Study of Quantitative Variables," Technometrics, Vol. 2, No. 4: page 455 (November 1960).
3. Box, G. E. P. and Draper, N. R. "Measures of Lack of Fit for Response Surface Designs and Predictor Variable Transformation," Technometrics, Vol. 24, No. 1: page 1 (February 1982).
4. Box, G. E. P. and Hunter, J. S. "Multi-Factor Experimental Designs for Exploring Response Surfaces," The Annals of Mathematical Statistics, Vol. 28, No. 1: page 195 (March 1957).
5. Bunnell, R. E. and Takacs, R. A. "Brik, An Interactive Goal Programming Model for Nuclear Exchange Problems," Unpublished Masters Thesis, Air Force Institute of Technology, (March 1984).
6. Cochran, William G. and Cox, Gertrude M. Experimental Designs (Second Edition). New York: John Wiley and Sons, Inc., 1964.
7. Cohen, Claude and Stein, Jack. Multi Purpose Optimization System users Guide Version 4. Evanston, Illinois: Vogelback Computing Center, Northwestern University (December 1978).
8. DeBaun, Robert M. "Response Surface Designs for Three Factors at Three Levels," Technometrics, Vol. 1, No. 1: page 1 (February 1959).
9. DeGroot, Morris H., Probability and Statistics, Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1975.
10. Devore, Jay L. Probability and Statistics for Engineering and the Sciences. Monterey, California: Brooks/Cole Publishing Company, 1982.

11. Dewispelare, Aaron R. and Sage, Andrew P. "On Combined Multiple Objective Optimization Theory and Multiple Attribute Theory for Evaluation and Choice Making," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-10, No. 6: page 315 (June 1980).
12. Fiacco, Anthony V. and McCormick, Garth P. Nonlinear Programming: Sequential Unconstrained Minimization Techniques, New York: John Wiley & Sons, Inc., 1968.
13. Hadley, G. and Whitin, T. M. Analysis of Inventory Systems, Englewood Cliffs, New Jersey: Prentice-Hall, 1963.
14. Hicks, Charles R. Fundamental Concepts in the Design of Experiments, New York: Holt, Rinehart, and Winston, Inc., 1964.
15. Ignizio, James P. Linear Programming in Single & Multiple-Objective Systems, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1982.
16. Karson, M. J. "Design Criteria for Minimum Bias Estimation of Response Surfaces," Journal of the American Statistical Association, Vol. 65, No. 332: (December 1970)
17. Karson, M. J. and others. "Minimum Bias Estimation and Experimental Design for Response Surfaces," Technometrics, Vol. 11, No. 3: (August 1969).
18. Keeney, Ralph L. and Raiffa, Howard. Decisions with Multiple Objectives: Preferences and Value Trade-Offs, New York: John Wiley and Sons, Inc., 1976.
19. Khuri, A. I. and Conlon, M. "Simultaneous Optimization of Multiple Responses Represented by Polynomial Regression Functions," Technometrics, Vol. 23, No. 4: page 363 (November 1981).
20. Kleinbaum, David G. and Kupper, Lawrence L. Applied Regression Analysis and Other Multivariable Methods, North Scituate, Massachusetts: Duxbury Press, 1978.
21. Lind, E. E. and others. "Fitting Yield and Cost Response Surfaces," Chemical Engineering Progress, Vol. 56, pp. 62-68 (1960).
22. Myers, Raymond H. Response Surface Methodology, Virginia Polytechnic Institute and State University, (1976).

23. Mylander, W. Charles and others. A Guide to SUMT - Version 4: Mclean, Virginia: Research Analysis Corporation.
24. Neter, John and Wasserman, William. Applied Linear Statistical Models, Homewood, Virginia: Richard D. Irwin, Inc., 1974.
25. Nie, Norman H. and others. Statistical Package for the Social Sciences (Second Edition), New York: McGraw-Hill, 1975.
26. Omah, Samuel D. "Regression Estimation for a Bounded Response Over a Bounded Region," Technometrics, Vol. 25, No. 3: (August 1983).
27. Protter, Murray H. Modern Mathematical Analysis, Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1964.
28. Quade, E. S. Analysis for Public Decisions (Second Edition), New York: Elsevier Science Publishing Company, Inc., 1983.
29. Smith, Palmer W. "A Methodology for Developing and Analyzing the Optimal Response of an Economic Criteria to Simultaneously Induced Events Using Input-Output, Mathematical Programming, and Response Surface Methodologies," Unpublished PhD Dissertation, University of Alabama, July 1975.
30. Smith, Palmer W. and Mellichamp, Joseph M. "Multi-dimensional Parametric Analysis using Response Surface Methodology and Mathematical Programming as Applied to Military Problems." Proceedings of the Pacific Conference on Operations Research, April 23-28, 1979, Seoul, Korea, Vol. I, pp. 592-615.
31. Sprent, Peter. Models in Regression, London: Methuen & Co. Ltd., 1969.
32. Vajda, S. The Mathematics of Experimental Design.
33. Wonnacott, Ronald J. and Wonnacott, Thomas H. Econometrics, New York: John Wiley and Sons, Inc.,
34. Zeleny, M. Multiple Criteria Decision Making, New York: McGraw-Hill, 1982.

Vita

Captain Robert E. Graney was born in Santa Barbara, California on January 13, 1949 and raised in Port Washington, New York. He received a Bachelor of Science Degree in Physics from Manhattan College in June, 1972. After receiving his commission, he entered undergraduate pilot training at Laughlin AFB, Texas. Subsequently, he was assigned to the SAC Airborne Command Post at Offutt AFB, Nebraska. After upgrading to aircraft commander, he flew with the 38th Strategic Reconnaissance Squadron also at Offutt AFB. Following his RC135 instructor pilot duties, Captain Graney returned to the Airborne Command Post as an aircraft commander. In 1980, he served as the 55SRW command control training officer and command post controller until his entry into the School of Engineering, Air Force Institute of Technology in August, 1982.

Permanent Address:
10 White Oak Drive
Port Washington, NY 11050

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; Distribution Unlimited		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GST/OS/84M-11			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
6a. NAME OF PERFORMING ORGANIZATION School of Engineering Air Force Inst. of Tech.		6b. OFFICE SYMBOL (If applicable) AFIT/ENS	7a. NAME OF MONITORING ORGANIZATION		
6c. ADDRESS (City, State and ZIP Code) Wright-Patterson AFB, OH 45433			7b. ADDRESS (City, State and ZIP Code)		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION		8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8c. ADDRESS (City, State and ZIP Code)			10. SOURCE OF FUNDING NOS.		
			PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.
11. TITLE (Include Security Classification) See Box 19			WORK UNIT NO.		
12. PERSONAL AUTHOR(S) Robert E. Graney, Captain, USAF					
13a. TYPE OF REPORT MS Thesis		13b. TIME COVERED FROM _____ TO _____		14. DATE OF REPORT (Yr., Mo., Day) 1984 March	
				15. PAGE COUNT 176	
16. SUPPLEMENTARY NOTATION <div style="text-align: right;">Approved for public release by E. WOLAVER Dean for Research and Professional Development Air Force Institute of Technology (ATO) Wright-Patterson AFB, OH 45433</div>					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB. GR.	Response Surface Methodology, Research Design, Regression Analysis, Multicriteria Optimization, Mathematical Programming.		
12	02				
19. ABSTRACT (Continue on reverse if necessary and identify by block number) Block 11 Title: AN OPTIMIZATION METHODOLOGY FOR MULTICRITERIA COMPARISON USING RESPONSE SURFACE METHODS AND MATHEMATICAL PROGRAMMING Thesis Advisor: Palmer W. Smith, LtCol, USAF.					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS <input type="checkbox"/>			21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a. NAME OF RESPONSIBLE INDIVIDUAL Palmer W. Smith, LtCol, USAF			22b. TELEPHONE NUMBER (Include Area Code) 513-255-3362		22c. OFFICE SYMBOL AFIT/ENS

END

FILMED

6-2-54

DTIC

